COMP 280 : Assignment 4

due: Thursday, February 17, 2000

1. (4 pts) Recall our skeleton program for checking equivalence of two expressions in propositional logic:

```
;; check-equiv-vars : formula formula vars → boolean
;; returns true iff the two formulas agree on their values for every
;; combination of values on the variables in vars. We assume that
;; vars contains all variables appearing in either input formula.
(define (check-equiv-vars F1 F2 vars)
  (cond [(empty? vars) (equal? (eval F1) (eval F2))]
        [else (and (check-equiv-vars (subst 'true (first vars) F1)
                                     (subst 'true (first vars) F2)
                                     (rest vars))
                   (check-equiv-vars (subst 'false (first vars) F1)
                                     (subst 'false (first vars) F2)
                                     (rest vars)))]))

;; eval : formula → boolean
;; reduces a formula containing only true and false to a boolean value
;;
;; subst : boolsym sym formula → formula
;; replaces every occurrence of sym in formula with boolsym
```

Modify *check-equiv-vars* to return true if the formulas are equivalent or a counterexample if the formulas are not equivalent. Be sure to provide all necessary data definitions. You do not need to type in and test your answer.

2. (4 pts) Express the negations of the following statements using quantifiers. Also express those negations in English.

   (a) Every student in this class likes owls.

   (b) There is a student in this class who has never been to Sammy's.

   (c) There is a student in this class who has taken every mathematics course offered at Rice.

   (d) Every student in this class has been in at least one room of every building on campus.

3. (6 pts) Provide a model and a countermodel for each of the following statements:

   (a) ∀ student ∃ prof : class-from(student, prof) → ∃ prof ∀ student : class-from(student, prof)

   (b) ∀ bar : foo(bar, baz(ball)) → ∀ bar : foo(bar, ball)

   (**Note:** The names in this problem are intentionally nonsensical to emphasize that the names used in formulas are irrelevant – only the interpretations matter[1].)

   For one point of extra credit, rewrite this statement with operations from arithmetic such that you'd expect the resulting formula to be true based on the names.

4. (2 pts) Consider the following statements:

   (a) $\forall x\, \forall y : P(x, y) \equiv \forall y\, \forall x : P(x, y)$

   (b) $\forall x\, \exists y : P(x, y) \equiv \exists y\, \forall x : P(x, y)$.

   Explain in words why the first is true but the second is false. Your justification may not refer to the fact that the first one has been given as an axiom.

[1] Though foo-bar-bazball is the name of the CS department's intramural softball team.

5. (2 pts) Prove the following axiom correct:

$$\exists x : (P(x) \vee Q(x)) \equiv \exists x : P(x) \vee \exists x : Q(x)$$

6. (2 pts) Is the following statement true or false? Justify your answer.

$$\exists x : (P(x) \wedge Q(x)) \equiv \exists x : P(x) \wedge \exists x : Q(x)$$

7. (3 pts) In class, we showed that to negate an expression with a single quantifier, we can replace it with the other quantifier and negate the predicate inside. This generalizes to arbitrary sequences of quantifiers. For instance:

$$\neg \forall x \, \exists y \, \exists z \, \forall w \; : P(x, y, z, w) \equiv \exists x \, \forall y \, \forall z \, \exists w \; : \neg P(x, y, z, w).$$

Prove this generalization by induction.