COMP 280 : Assignment 3, sample solutions

1. If q = emptyQ, then the axiom becomes:

   delQ(addQ(a, emptyQ)) = tail(putLast(a, emptyQ)) = tail(a :: emptyQ) = emptyQ

   If q $\neq$ emptyQ, then the axiom becomes:

   delQ(addQ(a, q)) = tail(putLast(a, q)) = tail(head(q) :: putLast(a, tail(q))) = putLast(a, tail(q)) = addQ(a, delQ(q))

2. We define the numbers as follow:

   - 0 is defined as emptyList.
   - succ(x) is defined as cons(emptyList, x).

   Then the number operations are implemented as follow, using the list operations:

   - zero?(x) is: EmptyL?(x).
   - succ(x) is: cons(emptyList, x).
   - pred(x) is: if EmptyL?(x) then emptyList else tail(x).

   We now prove that the axioms for numbers are true for the number operations so defined:

   - zero?(0) = EmptyL?(emptyList) = true.
   - zero?(succ(x)) = EmptyL?(cons(emptyList, x)) = false.
   - if succ(x) = succ(y), then cons(emptyList, x) = cons(emptyList, y), then tail(cons(emptyList, x)) = tail(cons(emptyList, y)), then x = y.
   - pred(0) = pred(emptyList) = if EmptyL?(emptyList) then emptyList else tail(emptyList) = if true then emptyList else tail(emptyList) = emptyList = 0.
   - pred(succ(x)) = pred(cons(emptyList, x)) = if EmptyL?(cons(emptyList, x)) then emptyList else tail(cons(emptyList, x)) = if false then emptyList else tail(cons(emptyList, x)) = tail(cons(emptyList, x)) = x.

3. If A, B, and C are false, then A $\rightarrow$ B is true, and (A $\rightarrow$ B) $\rightarrow$ C is false, but A is false so A $\rightarrow$ (B $\rightarrow$ C) is true.

4. $A \wedge B \rightarrow C$
   $\equiv \neg(A \wedge B) \vee C$ (conversion of $\rightarrow$)
   $\equiv (\neg A \vee \neg B) \vee C$ (De Morgan)
   $\equiv (\neg A \vee \neg B) \vee C \vee C$ (disjunction)
   $\equiv (\neg A \vee C) \vee (\neg B \vee C)$ (commutativity)
   $\equiv (A \rightarrow C) \vee (B \rightarrow C)$ (conversion or $\rightarrow$, twice).

5. $A \rightarrow B \vee C$
   $\equiv \neg A \vee (B \vee C)$ (conversion of $\rightarrow$)
   $\equiv \neg A \vee \neg A \vee (B \vee C)$ (disjunction)
   $\equiv (\neg A \vee B) \vee (\neg A \vee C)$ (commutativity)
   $\equiv (A \rightarrow B) \vee (A \rightarrow C)$ (conversion or $\rightarrow$, twice).

6. Let $W = A \vee B \rightarrow B = \neg(A \vee B) \vee B = (\neg A \wedge \neg B) \vee B$.

   Then:

   $W(A/true) = (\neg true \wedge \neg B) \vee B = (false \wedge \neg B) \vee B = false \vee B = B$.

   $W(A/false) = (\neg false \wedge \neg B) \vee B = (true \wedge \neg B) \vee B = \neg B \vee B = true$.

   Then:

$W(A/true, B/true) = true.$

$W(A/true, B/false) = false.$

So two leaves are true and one is false. The formula is therefore a contingency.

7. $(A \vee B) \wedge (C \rightarrow D)$
$\equiv (A \vee B) \wedge (\neg C \vee D)$ (conversion of $\rightarrow$)
$\equiv ((A \vee B) \wedge \neg C) \vee ((A \vee B) \wedge D)$ (distributivity)
$\equiv ((A \wedge \neg C) \vee (B \wedge \neg C)) \vee ((A \wedge D) \vee (B \wedge D))$ (distributivity, twice)
$\equiv (A \wedge \neg C) \vee (B \wedge \neg C) \vee (A \wedge D) \vee (B \wedge D).$