COMP 280 : Assignment 2 - Sample Solutions

1.
   - $1 \in S_2$
   - If $x \in S_2$, then $2x \in S_2$
   - $1 \in S_3$
   - If $x \in S_3$, then $3x \in S_3$
   - $S = S_2 \cup S_3$

2. (a) If $PS(S)$ represents the powerset of S:
   - $\emptyset \in PS(S)$
   - If $x \in S$ and $T \in PS(S)$, then $(\{x\} \cup T) \in PS(S)$

   (b) Formula: $|PS(S)| = 2^{|S|}$.
   Proof, by induction on the size of S:
   Base case: assume S is empty. Then $PS(S) = \{\emptyset\}$. So $|PS(S)| = 1 = 2^0 = 2^{|S|}$.
   Inductive step: assume $|PS(S)| = 2^{|S|}$. If $T = S \cup \{x\}$, and $x \notin S$, let's prove that $|PS(T)| = 2^{|T|}$.
   Suppose that, to create $PS(T)$, we first use the first axiom above to add the empty set, then go through all the elements of S in turn using each time the second axiom, then finally consider x and use the second axiom one last time for it.
   Just before considering x, the powerset of T will contain the same sets as the powerset of S, because all and only the elements of S will have been considered so far, plus the empty set, just as for the powerset of S. Now we consider x. For each set $U \in PS(T)$, we will create a new set $\{x\} \cup U$, and add it to $PS(T)$. So the number of sets in $PS(T)$ will double. So:
   $PS(T) = 2 * PS(S) = 2 * 2^{|S|}$ (by the induction hypothesis), so $PS(T) = 2^{|S|+1}$. But $T = S \cup \{x\}$ and $x \notin S$, so $|T| = |S| + 1$, which proves $PS(T) = 2^{|T|}$, and completes the proof.

3. (a) If S is a stack, then, by definition of a stack, S is either emptystack or push(e, S') for some e and some stack S'.
   If S is emptystack, then, by the fifth axiom, emptystack?(S) is true. Then either emptystack?(S) or nonemptystack?(S) is true (since emptystack?(S) is true, which is enough to prove the whole statement true, regardless of what nonemptystack?(S) might be).
   If S is push(e, S'), then, by the sixth axiom, nonemptystack?(S) is true. Which is again enough to prove the statement to be true, using the existing axioms.

   (b) No axiom directly tells the value of nonemptystack(newstack()). By definition of newstack, newstack() is equal to emptystack. So the proposed axiom is equivalent to nonemptystack(emptystack) = false. But no axiom tells the value of nonemptystack(emptystack), and no operation can be further applied. So the proposed axiom can not be proved using the existing axioms.

4. For some database D, year Y, and number N:

   - rank_of(first_class(D) D) = 1
   - rank_of(second_class(D) D) = 2
   - rank_of(third_class(D) D) = 3
   - rank_of(Y record_contrib(Y N $\emptyset$)) = 1
   - If rank_of(Y1 D) < rank_of(Y2 D), then amount_contrib(Y1 D) > amount_contrib(Y2 D), else amount_contrib(Y1 D) < amount_contrib(Y2 D).
   - If amount_contrib(Y1 D) < amount_contrib(Y2 D), then rank_of(Y1 D) > rank_of(Y2 D), else rank_of(Y1 D) < rank_of(Y2 D).
   - amount_contrib(Y record_contrib(Y N D)) = N
   - record_contrib(Y amount_contrib(Y N D) D) = D
   - record_contrib(Y1 N1 record_contrib(Y2 N2 D)) = record_contrib(Y2 N2 record_contrib(Y1 N1 D)).