COMP 280 : Assignment 2

due: Thursday, February 3, 2000

1. (2 pts) Provide an inductive definition of the set of all powers of 2 and all powers of 3 (*i.e.* $\{1, 2, 3, 4, 8, 9, 16, \ldots\}$).

2. (4 pts) Given a set S, the powerset of S is the set of all subsets of S. For example, if $S = \{1, 2\}$, then the powerset of S is $\{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$.

   (a) (2 pts) Provide an inductive definition of the powerset of a set S.

   (b) (2 pts) Determine a formula for the size of the powerset of S in terms of the size of S. Prove your formula correct using induction.

3. (4 pts) In class we discussed a model of stacks as follows:

   stack[alpha] is the following inductively defined set:

   - emptystack is in stack[alpha]
   - if S is in stack[alpha] and e is in alpha, push(e, S) is in stack[alpha]

   nonemptystack[alpha] is the set of all push(e, S) where S is in stack[alpha].

   Operations:

   - top : nonemptystack[alpha] → alpha
   - pop : nonemptystack[alpha] → stack[alpha]
   - push : alpha, stack[alpha] → nonemptystack[alpha]
   - newstack : → emptystack
   - emptystack? : stack → boolean
   - nonemptystack? : stack → boolean

   Axioms:

   - top(push(e,S)) = e
   - pop(push(e,S)) = S
   - push(top(S), pop(S)) = S
   - emptystack?(push(e,S)) = false
   - emptystack?(emptystack) = true
   - nonemptystack?(push(e,S)) = true

   Your friend has come up with two new axioms about stacks:

   (a) For all stacks S, either emptystack?(S) or nonemptystack?(S) is true.

   (b) nonemptystack?(newstack()) = false.

   Determine whether your friend's proposed axioms are necessary. In other words, for each one, either prove that it is true based on the existing axioms, or argue (briefly!) why you cannot prove it based on the existing axioms.

4. (5 pts) The Alumni Office stores information about how much money each graduating class has contributed to the university in the past year. Their data model (database) is a list of tuples. Each tuple contains the class year and the amount of money contributed (in thousands of dollars). The database is sorted in decreasing order on the contributions. For example:

$$\langle\langle 1985, 300\rangle, \langle 1989, 250\rangle, \langle 1999, 12\rangle\rangle$$

Their model uses the following operators:

- rank_of : year database → number
  Given a class, return its position (1-based) in the ranking
- record_contrib : year amount → database
  Given a class and an amount, record the class' contribution
- first_class : database → year
  Returns the class in the first position in the database
- second_class : database → year
  Returns the class in the second position in the database
- third_class : database → year
  Returns the class in the third position in the database
- amount_contrib : year database → number
  Returns the amount associated with the given class in the database

Propose a set of axioms on these operators. Your axioms should characterize the requirements on the operators as fully as possible. However, you should not propose axioms that you could prove true using other axioms.