COMP 280 : Assignment 11

due: Thursday, April 20, 2000

1. (2 pts) Construct a finite automaton that accepts all sequences over $\Sigma = \{0, 1\}$ that contain an even number of 1's.

2. (2 pts) Construct a finite automaton over $\Sigma = \{0, 1\}$ that accepts all sequences containing no subsequence of at least 3 consecutive 0's.

3. (3 pts) A (simplified) barcode is a sequence of 12 binary digits. The first two digits form the header, the last two the trailer, and the middle eight digits are the actual barcode data. Barcodes may be scanned either forwards or backwards. When scanned forwards, the header digits are 00 and the trailer digits are 10. When scanned backwards, the header digits are 01 and the trailer digits are 00. There are no restrictions on the data digits.

   Is it possible to construct a FA (in which the transition relation R is a function) that recognizes barcodes? Justify your answer (if you claim it is possible, you must provide the relevant FAs to backup your claim).

4. (4 pts) Construct a FA with two inputs and one output that adds two binary numbers of the same length. The numbers to add are the inputs; the output is the sum (also in binary). Assume that you receive the input bits in reverse order. In other words, the sequence $\langle 0, 0, 1 \rangle$ is the input corresponding to 4.

5. (5 pts) Let $A = \langle Q, \Sigma, q_0, R, F \rangle$ be a finite automaton. Prove that if R is a function from $Q \times \Sigma \rightarrow Q$, then for all non-empty input sequences x and y and all states q, $R^*(q, xy) = R^*(R^*(q, x), y)$ [Hint: use induction on the length of y.]

6. (8 pts) We can model the controller for a traffic light as a pair of finite automata, one controlling the light in each direction (*i.e.*, one for the north-south lights and one for the east-west lights at an intersection). Each traffic light FA monitors two inputs: one indicating whether a car is waiting at that light, and the other indicating the current color of the light in the other direction. Each FA outputs the current color of the light that it controls.

   Well-designed traffic lights help prevent accidents. Call a traffic light safe if it can never output green lights in both directions at the same time.

   (a) (2 pts) Provide two finite automata that model the execution of a traffic light.

   (b) (3 pts) State a condition under which your traffic light design is safe. Your condition should be mathematically precise and testable. In other words, saying "both lights aren't green at once" is insufficient (becuase that doesn't indicate how to check it). Express your condition in terms of the automata that you defined to model the traffic light.

   (c) (3 pts) Implement your traffic light design as a Prolog program (similar to the soda machine example that we discussed in class). Use your program to test whether your design is safe.