CS503
Homework #4

I worked with:

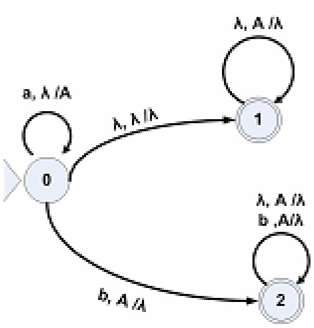
I consulted:

#1. a) Given the following PDA, M:

$Q = \{q_0, q_1, q_2\}$
$\Sigma = \{a,b\}$
$\Gamma = \{A\}$
$F = \{q_1, q_2\}$

$\delta(q_0,a,\lambda) = \{[q_0,A]\}$
$\delta(q_0, \lambda,\lambda) = \{[q_1, \lambda]\}$
$\delta(q_0,b,A) = \{[q_2, \lambda]\}$
$\delta(q_1, \lambda,A) = \{[q_1, \lambda]\}$
$\delta(q_2, b,A) = \{[q_2, \lambda]\}$
$\delta(q_2, \lambda,A) = \{[q_2, \lambda]\}$

a) Draw the graph for M



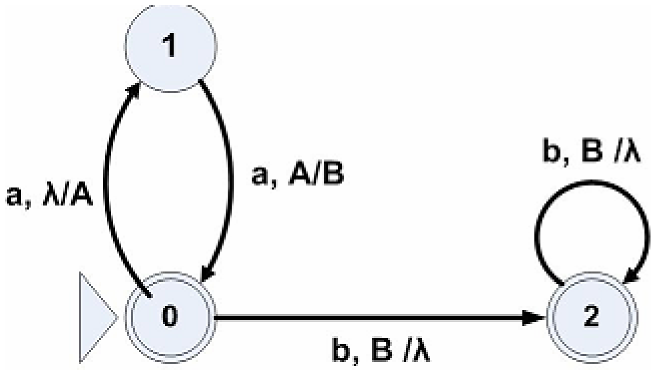
b) Trace the computations of *aab, abb, aba, aabb*

[q0,aab,λ] → [q0,ab,A] → [q0,b,A] → [q2, λ, λ] (accepted)
[q0,abb,λ] → [q0,bb,A] → [q0,b, λ] → [q1, b , λ] (rejected)
[q0,aba,λ] → [q0,ba,A] → [q0,a, λ] → [q1, a , λ] (rejected)
[q0,aabb,λ] → [q0,abb,A] → [q0,bb,AA] → [q2,b,A] → [q2, λ, λ] (accepted)

b) What is L(M)?

$L(M) = \{a^n b^m \mid n \geq m \geq 0\}$

#2. a) Construct a PDA to accept $\{a^n b^{2n} \mid n \geq 0\}$



b) Show computations on *a a b* and *a b b*

[q0,aab,λ] → [q1,ab,A] → [q0,b,B] → [q0, λ, λ] (accepts)

[q0,abb,λ] → → [q1,bb,A] → halts (rejects)

#3. Show context free languages are closed under reversal. Show your method on $\{ab^n \mid n \geq 0\}$

If L is a CFL, then there is a grammar, G, with L = L(G).
For any production, A → α in G, create a new grammar with
$A \rightarrow \alpha^R$

For L = $\{ab^n \mid n \geq 0\}$, G is

S → a A | a
A → b A| b

and G for $L^R = \{ba^n \mid n \geq 0\}$, is

S → A a | a
A → A b | λ

#4. a) Given G is in Chomsky Normal form, prove using induction that length(derivation) = 2n-1 when |w| = n

Proof by induction on length(derivation)

Basis |w|

If |w| = 1, w = a ε Σ and a is derived in a derivation of length 1: S → a

length(derivation) = 1, |w| = 1 and 2 * 1 – 1 = 1, the length of w.

Induction Hypothesis

Assume length(derivation) = 2n-1 when $1 \le |w| \le n$
<u>Induction Hypothesis</u>

Show: length(derivation) = 2(n+ 1)-1 = 2n+1 when $|w| = n + 1$

Since $n \ge 1$, $n + 1 \ge 2$, so the derivation must start S ➔ A B for some variables A, B. then, the entire derivation is S ➔ A B ➔ $a_1 a_2 \ldots a_k \; b_1 \, b_1 \ldots b_{(n+1)-k}$ where

Note that neither A nor B can derive the null string.

A ➔ $a_1 a_2 \ldots a_k$ and B ➔ $b_1 \, b_1 \ldots b_{(n+1)-k}$

Since $k \le n$, length(derivation of $a_1 a_2 \ldots a_k$) = 2k-1
and since $(n+1) - k \le n$
length(derivation of $b_1 \, b_1 \ldots b_{(n+1)-k}$) = 2(n+1-k) -1 by the induction hypothesis

Thus, length(derivation) of w from AB = 2k-1 + 2(n+1-k) -1 = 2n. Adding on S➔ A B gives length(derivation) = 2n+1.

#5. Convert your grammar for L from problem #3 above to a PDA using the technique in the book. Show both a derivation and a computation of *a b b*

(Note: If your grammar is not in GNF, convert it – should be easy to do this)

δ(q0,a,λ) = [q1,A]
δ(q1,b,A) = [q1,A]
δ(q0,a, λ) = [q1, λ]
δ(q1,b, A) = [q1, λ]

<u>Derivation</u>

S ➔ a A ➔ a b A ➔ a b b

<u>Computation</u>
[q0,abb,λ] → [q1,bb,A] → [q1,b,A] → [q1, λ,λ]