Homework #3

1. **True or False:**

a) *a a b a* matches *a* + b** T F

b) *babab* matches *b (ab)** T F

c) If A = Φ then A B = Φ for all languages B T F

d) If A = {ε} then A B = Φ for all languages B T F

e) If A = a* and Σ = {a,b}, then Σ* - A = b* T F

#2. Write regular expressions for the set of strings of 0's and 1's with at most one pair of consecutive 1's

Exercise says "at most one" occurrence of "11", so start with

(ε + 1 + 11)

Now build around it the combination of 1's and 0's that will prevent another occurrence of "11":

In front:

(0 +10)*

After:

(0 + 01)*

So altogether: (0 +10)* (ε + 1 + 11) (0 +10)*

#3. Draw the graph for the following DFA and then convert to regular expression.

	0	**1**
* → p	s	p
q	p	s
r	r	q
s	q	r

This gives you an idea how messy these are, and how you'd really like to plug them into a computer program!

Book's method, removing q

$\alpha_{pp}^{pqrs} = \alpha_{pp}^{prs} + \alpha_{pq}^{prs} (\alpha_{qq}^{prs})^* \alpha_{qp}^{prs}$

$\alpha_{pp}^{prs} = 1^*$

α_{pq}^{prs} = 1*0 (0 +10*1)

α_{qq}^{prs} = 01*0 (0 + 10*1) + 1 (0 +10*1)

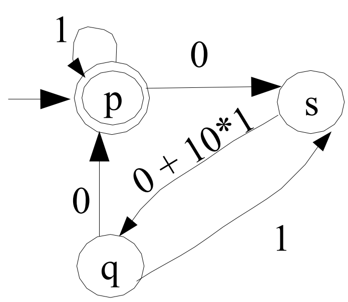
α_{qp}^{prs} = 01*

So α_{pp}^{pqrs} = 1* + 1*0 (0 +10*1) (01*0 (0 + 10*1) + 1(0+10*1)* 01*

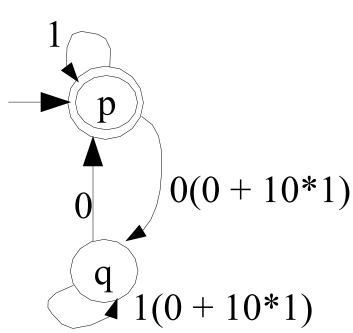
I'm sure this can be simplified. If anyone wants to send me their simplification, I'll post it with full credit to the sender!

Other techniques Other texts have us eliminate states until we just have an initial and a final state (same here) left:

Step 1:
Eliminating "r":

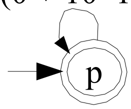


Step 2 Then eliminating "s":



Step 3: Eliminating "q"

1+ (0(0 + 10*1))(1(0 + 10*1))*0



Now we * the expression on the loop to get the regular expression:

(1 + (0(0 + 10*1))(1(0 + 10*1))*0)*

Are these two the same? I hope so, and again, using the equivalencies and many hours of our time, we could probably show it.

#4. Does (R +S)* S = (R*S)* Justify your answer

Counter-example: if ε is not in S, then they are not equal.

#5 - #6. Given R is a regular language and N is a non-regular language

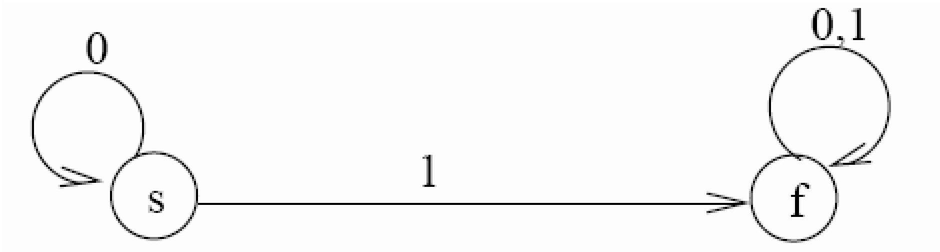
#5. (5 Points) Suppose X is a language such that N = ~X (~means complement). Does it follow that X must be regular? If so, state why. If not, does it follow that X must be non-regular? If so, state why. If neither of these is true name 1) a specific non-regular N such that N = ~X with X non-regular and 2) a specific non-regular N satisfying N = ~X with X regular.

X cannot be regular because then its complement would be regular and it is given that N is not regular.

#6. (5 Points) Suppose X is a language such that X = R ∩ N. Does it follow that X must be regular? If so, state why. If not, does it follow that X must be non-regular? If so, state why. If neither of these is true name 1) a specific non-regular N and a regular R such that X = R ∩ N with X non-regular and 2) a specific non-regular N and a regular R satisfying X = R ∩ N with X regular.

Neither is true:
1) Let R = 0*1* and N = 0^n1^n. Then the intersection is 0^n1^n which is non-regular.
2) Let R = φ and N = 0^n1^n. Then the intersection is φ which is regular.

#7. Create a dfa to accept (0+1)*1(0+1)*


0
0,1
1
s
f