**Name ________________________**

**Homework 9**

Worked with ______________

Consulted: ________________

1. For each of the language classes: *regular, context-free, recursively enumerable,* and *recursive*, and each operation: *complement, union, intersection, concatenation,* and *closure, indicate whether the class is closed under that operation by filling in the corresponding spot in the following table with “yes” or “no”.

| | regular | context-free | recursively enumerable | recursive |
|---|---|---|---|---|
| complement | | | | |
| union | | | | |
| intersection | | | | |
| concatenation | | | | |
| *-closure | | | | |

#2. Suppose $L_1$, $L_2$, and $L_3$ are recursively enumerable languages over the alphabet $\Sigma$ such that
(a) $L_i \cap L_j = \Phi$ when $i \neq j$; and
(b) $L1 \cup L2 \cup L3 = \Sigma^*$

Prove that $L_1$ is recursive. Indicate clearly how each of the hypotheses (a) and (b) are used in your argument.

#3. Suppose $L$ is a recursive language. Prove that $L^*$ is recursive. Write this out in words rather than using pictures.

#4. Complete each sentence with one of the answers:

- *regular*
- *context-free*
- *Recursive*
- *Recursively enumerable*
- *none of the above*

Make the strongest true assertion you can. In each case you should explain your answer by reference to the appropriate closure properties of the relevant language classes. ($B^c$ *means the complement of B)*

(a) If $A$ is regular and $B$ is regular, then $A \cup B^c$ is:

(b) If $A$ is context-free and $B$ is regular, then $A \cup B^c$ is:

.

(c) If $A$ is regular and $B$ is context-free, then $A \cup B^c$ is:

(d) If $A$ is recursive and $B$ is recursive, then $A \cup B^c$ is: *recursive.*

(e) If $A$ is recursive and $B$ is Recursively enumerable, then $A \cup B^c$ is:

(f) If $A$ is recursively enumerable and $B$ is recursive then $A \cup B^c$ is:

#5. Show the smallest category (regular, c-f, recursive, re, non-re) that each of the following is in. Prove that the language is in that category and prove that it is not in the next innermost category.

a) L(G) where G:

$$S \rightarrow aB \mid b\,A$$
$$A \rightarrow a\,S \mid b\,A\,A \mid a$$
$$B \rightarrow b\,S \mid a\,B\,B \mid b$$

b) L(G) where G:

$$S \rightarrow a\,S \mid b\,S \mid \varepsilon$$

c) $L_1 \cap L_2$ where $L_1$ and $L_2$ are context-free

d) The set of encodings of Turing Machines M whose time complexity is not bounded by $n^2$; i.e., L = {M | there exists an input string w such that M performs more than $|w|^2$ steps on input w}

#6. Consider the following language:

HALT = {< *M, w* > | *M* is a Turing machine and $w \in L(M)$}

*Here < M,w > denotes some encoding of the pair consisting of a description of machine M and of input w, so that HALT is an encoding of the set of machine-input pairs such that the machine accepts the input. The specific details of the encoding itself are irrelevant to the questions which follow.*

The theorem on the undecidability of the halting problem states that HALT is not decidable. For this problem, take that theorem as given.

(a) Is the language HALT recursively enumerable? Defend your answer.
(b) Is the complement of HALT recursively enumerable? Defend your answer.

#7. Decidable or undecidable: whether a language is regular or not. Prove your answer. (Hint: Look at $Reg_{TM}$ = {M | M is a TM and L(M) is regular})

#8. Show that there is no algorithm that determines whether an arbitrary Truing machine prints a 1 on its final transition.

#9. Show whether the following instance of PCP has a solution or not. (01,011), (001,10), (10,00)

#10. Explain the differences between NP, NP-Complete and undecidable briefly but clearly (I know we have not covered this in class)