CS 521, HW 2: Natural Deduction Proofs

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Due Tuesday, 7 September 2010

I. A Natural Deduction Derivation Step-by-Step. Prove

$$\vdash\ ((p \vee q) \to r) \to (p \to r) \tag{1}$$

To do so, start with two instances of the Axiom rule:

$$(p \vee q) \to r,\ p\ \vdash\ p \quad \text{and} \quad (p \vee q) \to r,\ p\ \vdash\ (p \vee q) \to r$$

You want to use $\vee$-introduction to produce $p \vee q$, $\to$-elimination to infer r, and $\to$-introduction to complete the proof.

Please also notice what you've proved: $p \vee q$ is a weaker hypothesis than p, since it's easier for $p \vee q$ to be true. The result says that if r follows from the weaker hypothesis $p \vee q$, then r also follows from the stronger hypothesis p.

II: Systematically Constructing Natural Deduction Proofs. Use the natural deduction inference rules in the lecture notes on "Consequence Relations and Natural Deduction"[1] to give derivations of the following judgments. In each case, work start at the bottom of your derivation, with the desired conclusion. Repeatedly choose an introduction rule that could produce this conclusion. When no introduction rule is relevant, look for an elimination rule that could produce your goal.

$$\vdash\ (p \wedge q) \to (q \wedge p) \tag{2}$$

$$\vdash\ (p \wedge q) \to (p \to q) \tag{3}$$

$$\vdash\ ((p \to \bot) \wedge q) \to (p \to q) \tag{4}$$

$$p \wedge (q \vee r)\ \vdash\ (p \wedge q) \vee (p \wedge r) \tag{5}$$

$$(p \wedge q) \vee (p \wedge r)\ \vdash\ p \wedge (q \vee r) \tag{6}$$

III. Propositions as Types. For each of the derivations you've just constructed, re-write it with explicit proof objects. Use the rules on pp. 16–19 in the Explicit Proof slides[2] to insert variables, pairs, etc. into the derivations you've just written.

[1] At URL `http://web.cs.wpi.edu/~guttman/cs521_website/consequence.pdf`.

[2] At URL `http://web.cs.wpi.edu/~guttman/cs521_website/printable_explicit_proofs_2sep10.pdf`.