Have a look at the translation exercises as posted at the course web site, and work through as many of them as you need to in order to get comfortable. Don't turn anything in. Ask questions in class Monday.

Exercises to submit

1. Refer to the formal definition of the formulas of propositional logic below. For any formula φ let $\mathbf{c}(\varphi)$ denote the number of occurrences of the binary connectives $\wedge$, $\vee$, and $\rightarrow$. Let $\mathbf{a}(\varphi)$ denote the number of occurrences of propositional atoms in φ.

   Make a conjecture about the relationship between $\mathbf{a}(\varphi)$ and $\mathbf{c}(\varphi)$.

   Prove your conjecture by induction on the length of φ.

   Yes, this result is intuitively clear once you look at two or three examples of formulas. But the point to this problem is to show how to do careful proofs by induction over formulas, which are an absolutely crucial technique. The majority of the results in the course are proved by induction over formulas, so let's make sure the structure of such a proof is clear to you....

2. Read the handout on Konig's Lemma (course web page) and do the exercises there.

3. A binary tree is a tree such that each node has either 0 or 2 children. Find a formula relating the number of leaf nodes and the total number of nodes in a finite binary tree T. Prove your answer by induction (on what? Make it clear).

Here is the official definition of propositional logic formula.

Definition. *Fix a set $\mathcal{A}$ of* atomic formulas.

The set $\mathcal{F}$ of formulas *is given inductively by*

- *Every atomic formula is a formula,*
- *If α is a formula then $\neg\alpha$ is a formula,*
- *If α and β are formulas then $\alpha \wedge \beta$ is a formula,*
- *If α and β are formulas then $\alpha \vee \beta$ is a formula,*
- *If α and β are formulas then $\alpha \rightarrow \beta$ is a formula.*

When we say that $\mathcal{F}$ is "given inductively" by the above what we mean is that $\mathcal{F}$ is the smallest set satifying those closure conditions.