

The Master Theorem

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Master Theorem: If $T(n) = a \cdot T(\lfloor n/b \rfloor) + f(n)$, and $v = \log_b a$, then:

1. If $f(n) \ll O(n^v)$, then $T(n) \in \Theta(n^v)$
2. If $f(n) \in \Theta(n^v)$, then $T(n) \in \Theta(n^v \log n)$
3. If $f(n) \gg \Omega(n^v)$, then $T(n) \in \Theta(f(n))$

Small print:

$f(n) \ll O(n^v)$ means $f(n) * n^\epsilon \in O(n^v)$ for some $\epsilon > 0$.
 $f(n) \gg \Omega(n^v)$ means $f(n) \in \Omega(n^{v+\epsilon})$ for some $\epsilon > 0$.
Case 3 has an extra condition that's rarely a problem.

See *CLRS*, Theorem 4.1, p. 94. We have slightly simplified the notation.