## CS 4732: <br> Computer Animation

## Orientation Representations

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## Euler Angles: Idea

$\square$ Represent object orientation as a sequence of rotations about $X, Y, \& Z$, ee.g., roll ( $z$ ) then pitch ( $x$ ) then yaw ( $y$ ). $\square O r$ as a rotation matrix:

$$
\left(\begin{array}{cccc}
c 2 c 3 & c 2 s 3 & -s 2 & 0 \\
s 1 s 2 c 3-c 1 s 3 & s 1 s 2 s 3+c 1 c 3 & s 1 c 2 & 0 \\
c 1 s 2 c 3+s 1 s 3 & c 1 s 2 s 3-s 1 c 3 & c 1 c 2 & 0 \\
0 & 0 & 0 & 1
\end{array}\right)
$$

- c1 \& s1 are the x-axis cos \& sin, etc.


## Euler Angles: Good \& Bad

## WPI

$\square$ Easy for us to understand/specify
$\square$ Gimbal lock

- Lose one DOF after some rotations
$\square$ How do we interpolate?


Global coordinate system


Local coordinate system attached to object

## Angle \& Axis Representation

$\square$ Going from one orientation to another can be represented as a single rotation about a single axis.


## Quaternions

$\square A$ quaternion is a scalar and a vector

- $\mathrm{q}=[\mathrm{s}, \mathrm{v}]$
$\square[s, v]=[-s,-v]$
$\square$ Addition:
$-[s 1, v 1]+[s 2, v 2]=[s 1+s 2, v 1+v 2]$
- Length:
$\square\|q\|$ of $[a,(b, c, d)]=\operatorname{sqrt}\left(a^{2}, b^{2}, c^{2}, d^{2}\right)$
$\square$ Multiplication:
-q1*q2 $=[s 1 s 2-v 1 \cdot v 2, s 1 v 2+s 2 v 1+v 1 \times v 2]$


## Quaternions (cont.)

$\square A$ series of rotations can be concatenated by multiplying them together.
$\square$ Multiplicative identity:

- [1, (0, 0, 0)]
$\square$ We can think of quaternions as being derived from Angle \& Axis notation.
-Interpolation of quaternions is covered next week (Sec. 3.3)

