CS 563 Advanced Topics in Computer Graphics
Nonlinear Projections

by Joe Miller

Nonlinear vs Linear Projections

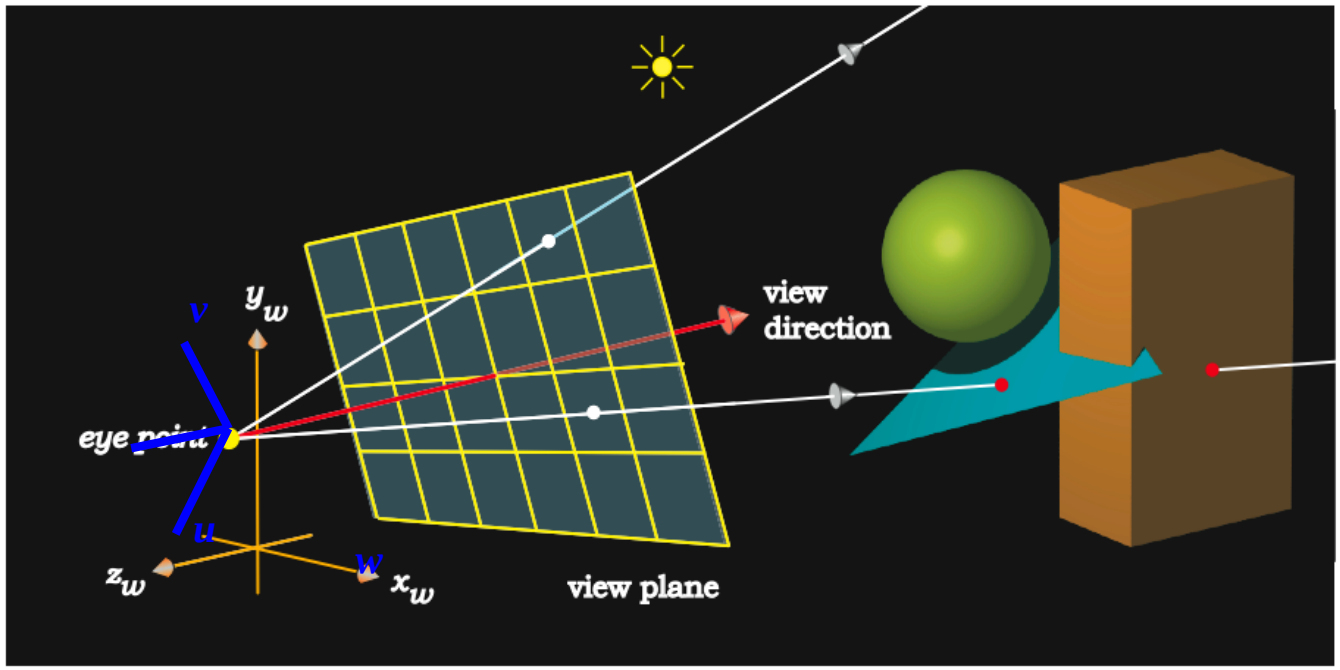
- What is a nonlinear projection?
  - Anything that's not a linear projection.

- *They are known as linear projections because the projectors are straight lines*

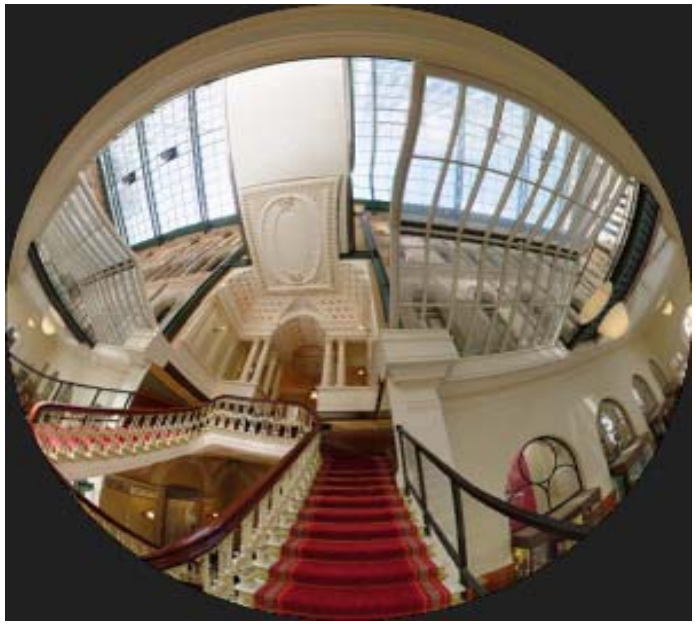
- A linear system is defined by system which satisfies the following properties
  - **Superposition** – $f(x + y) = f(x) + f(y)$
  - **Homogeneity** – $f(kx) = k*f(x)$

Pinhole camera review

- Recall the pinhole camera algorithm
- This technique casts a ray in a direction defined by the camera through the view plane into the scene to create the projection
  - $\boldsymbol{d} = x_v \boldsymbol{u} + y_v \boldsymbol{v} - d\boldsymbol{w}$

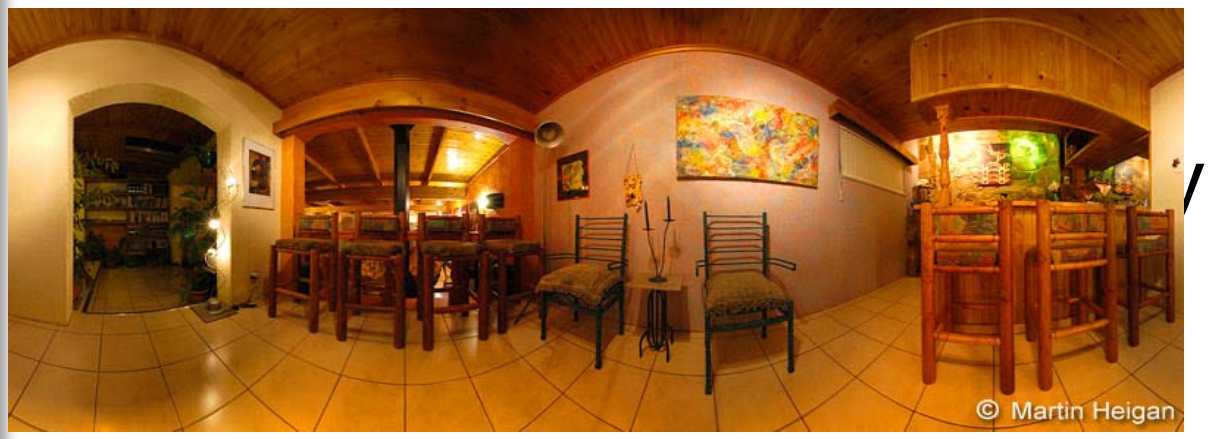


Examples of Nonlinear Projections



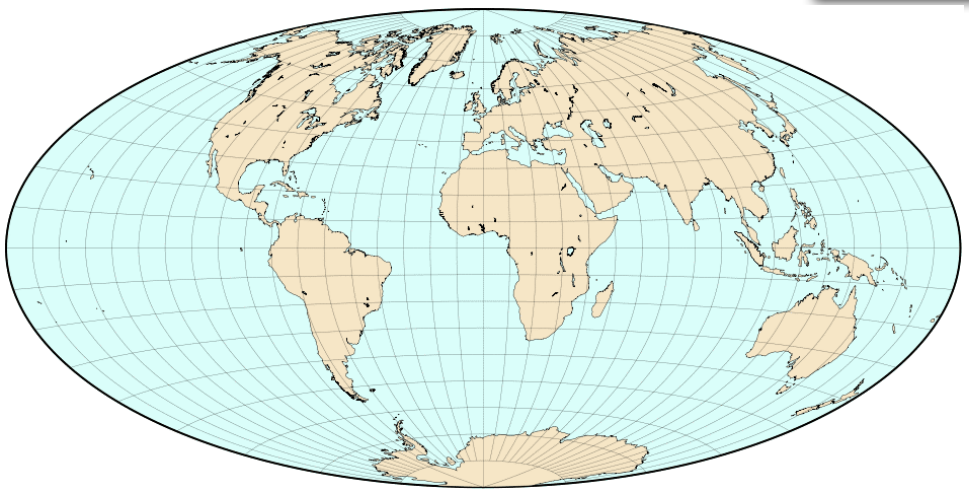
Fisheye

Spherical Panoramic



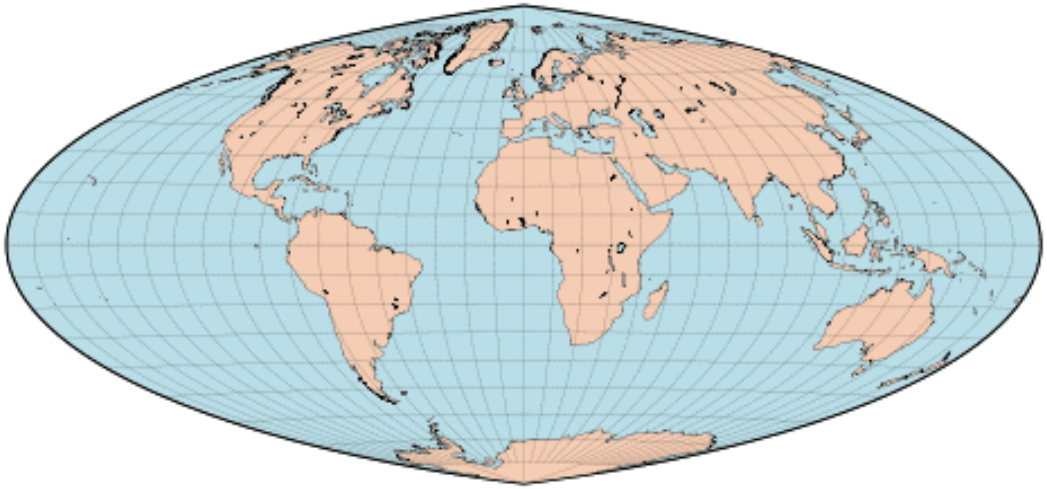
ylindrical Panoramic

More Projections



Aitoff Projection

Bonne Projection



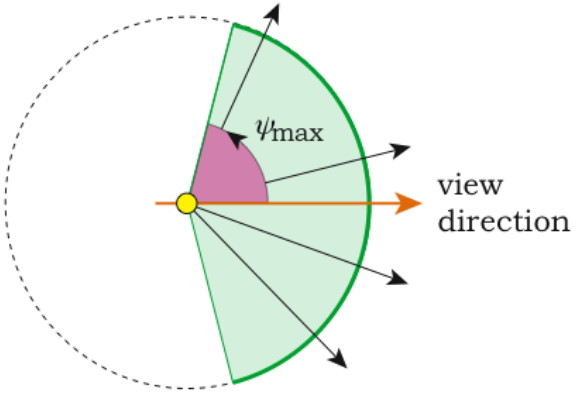
Eckert-Greifendorff Projection

Fisheye Lens

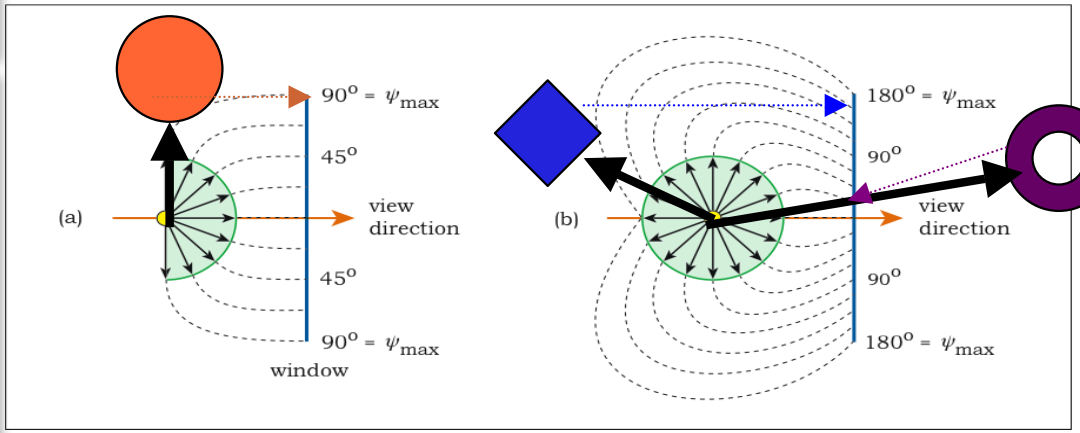
- Fisheye is not just a computer projection but a physical technique used in photography
- Fisheye lens are super wide angle lens which can capture up to 180° field of view.
- Originally designed for scientific studies
  - Primary use was for astrophotography and astronomical observations
  - Once called the “full sky lens”
- Designed to mimic that of a fish looking through water

Fisheye Projection

- Ray tracing mimics real life by projecting what the lens would see onto the view plane.



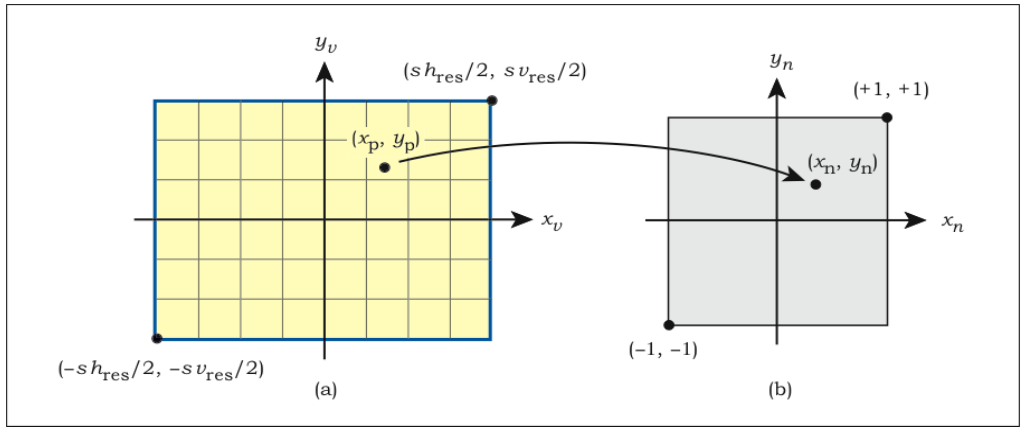
- Define the camera by the maximum it can see – the half angle or ψ_{max}
- The half angle will define the fov → fov = $2\psi_{max}$



- As before cast rays from the camera to objects
- Extract color information from the objects and paint them on the viewport

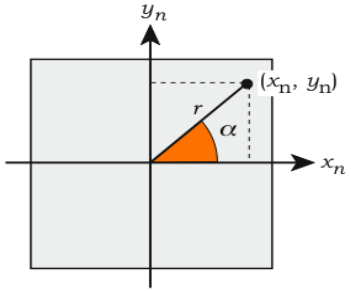
Fisheye Projection

- Here are details on how the system projects the scene onto display
- Normalize the view plane into a unit square

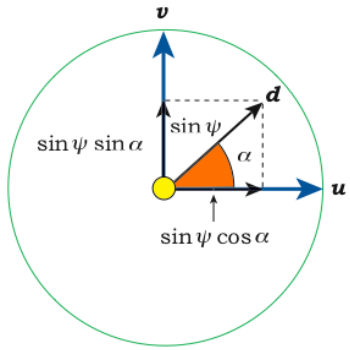


- $x_n = x_p*(2/(s*h_{res}))$
- $y_n = y_p*(2/(s*v_{res}))$

- Convert normalized coordinates to polar coordinates in terms of (r, ⟨)
  - sin ⟨ = y_n / r
  - cos ⟨ = x_n / r

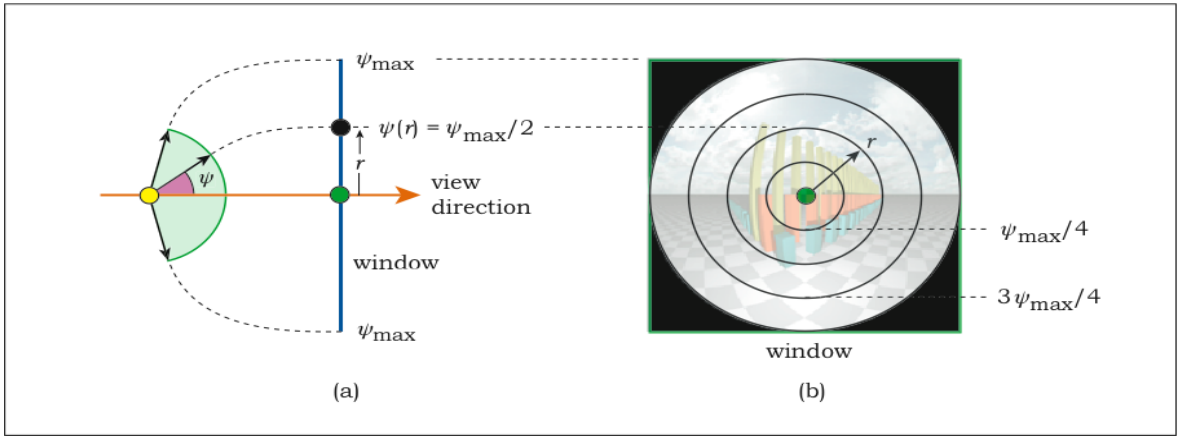


- If r<=1 then this defines the ***uv*** plane in terms of a circle



Fisheye Projection

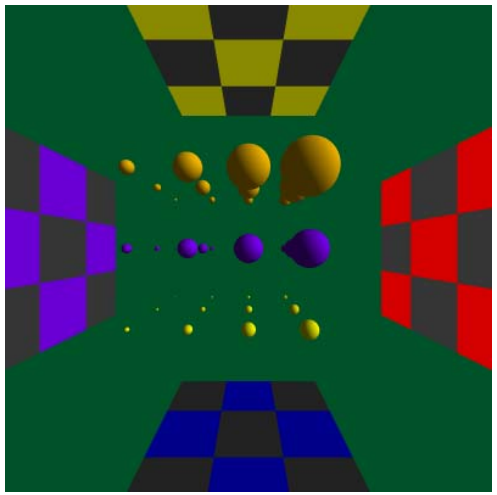
- Uniformly project the rays through uv plane by keeping angular distance between them the same
  - $\psi = r * \psi_{max}$



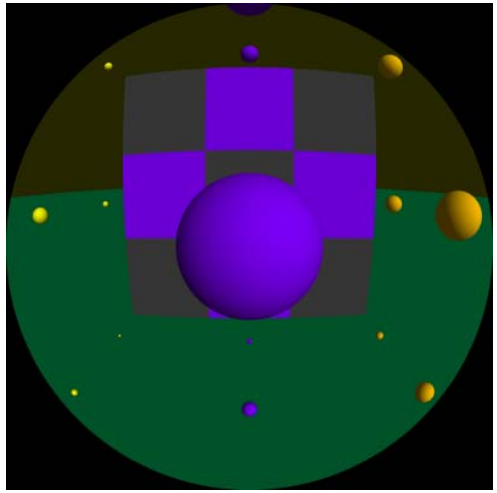
- Convert the spherical coordinate (θ, ψ) to cartesian coordinates to define the ray.
  - $\boldsymbol{d} = \sin\psi\cos\theta\,\boldsymbol{u} + \sin\psi\sin\theta\,\boldsymbol{v} - \cos\psi\,\boldsymbol{w}$
- Where did spherical coordinates come from?
- What happened to r?
- Why is the cos negative?

Fisheye Projection Examples

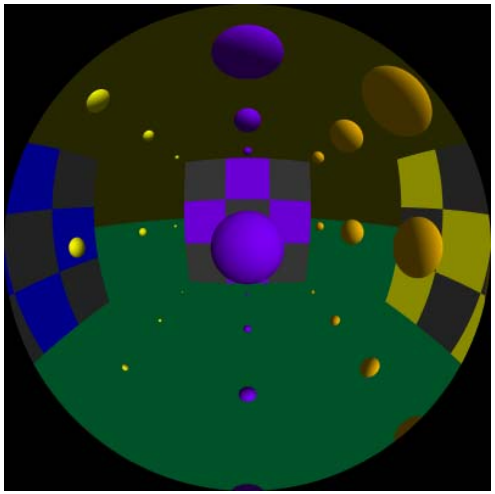
- Some examples of fish eye projections



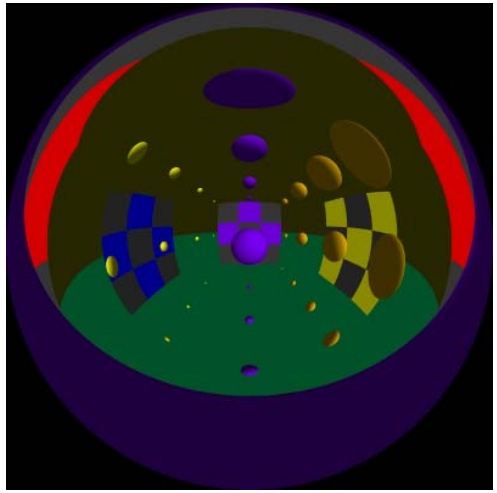
Pinhole of the scene



45° fisheye



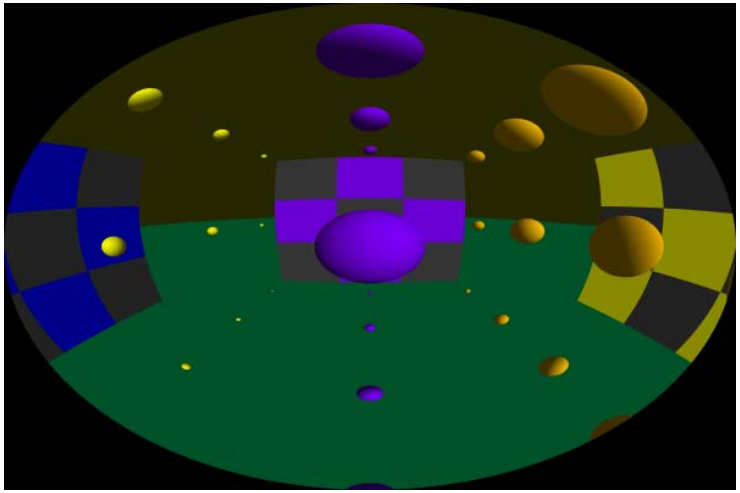
90° fisheye



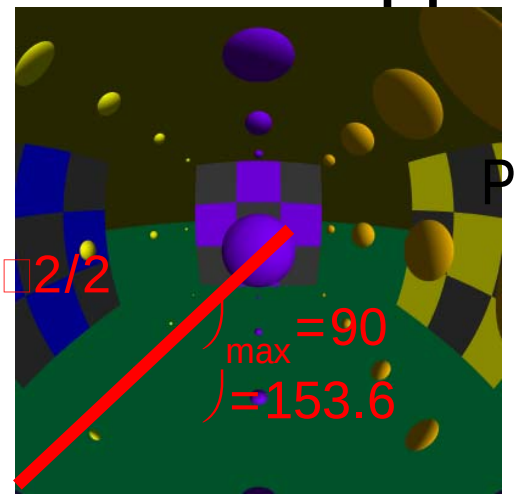
180° fisheye

Fisheye Projection Extras

- Stretching the aspect ratio

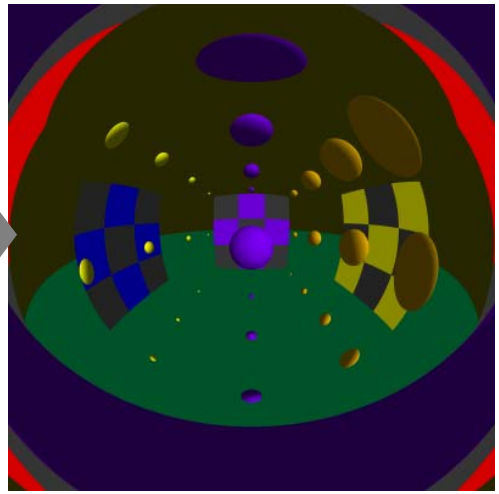


- What happens if the r<=1 were removed?



Problems when ϕ > 180

ϕ_{max} = 150

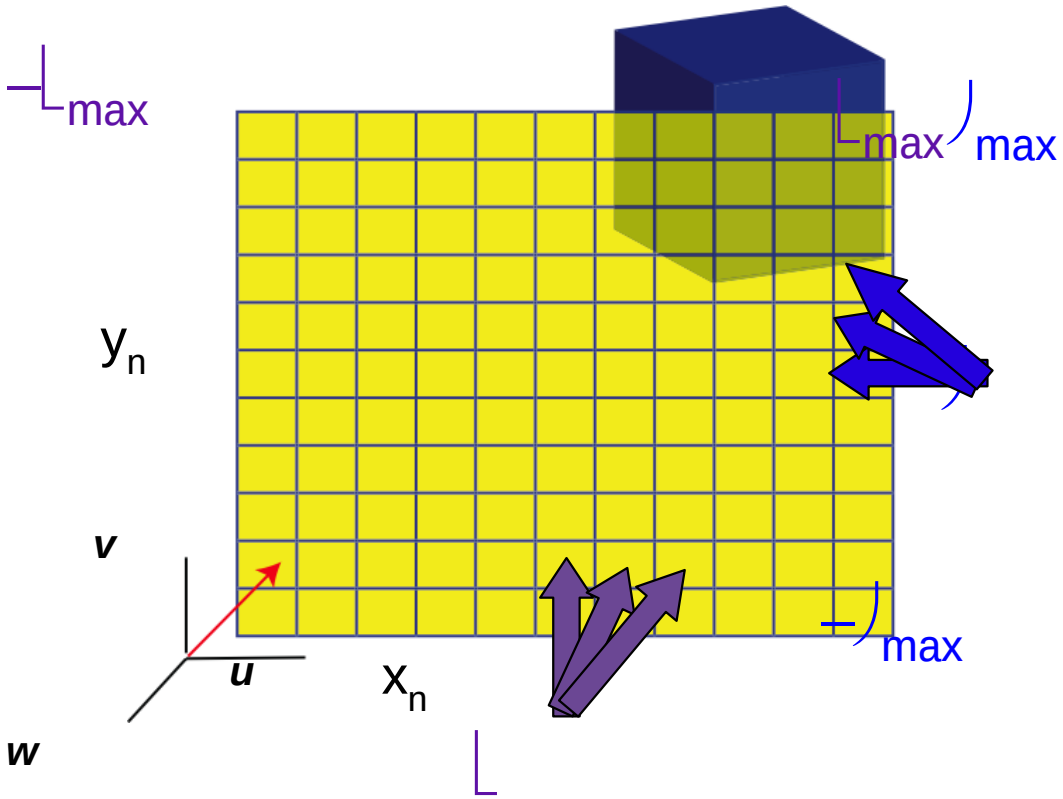


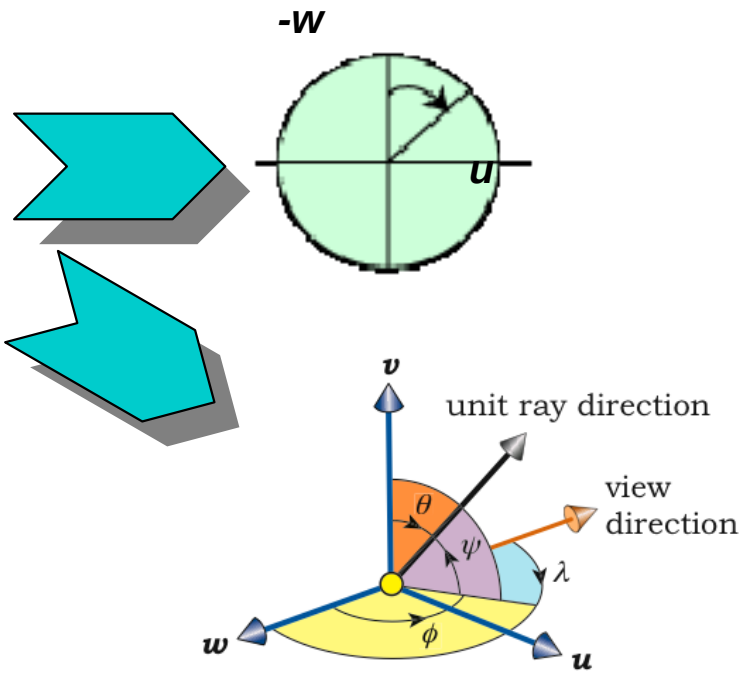
Spherical Panoramic Projection

- Another type of non-linear projection which can be simulated with ray tracing is the spherical panoramic projection
- Like the fisheye projection, spherical panoramic projections have an application in the real world
  - Photographers take multiple rows of pictures and stitch them together with software to produce an image covering a large field of view

Spherical Panoramic Projection

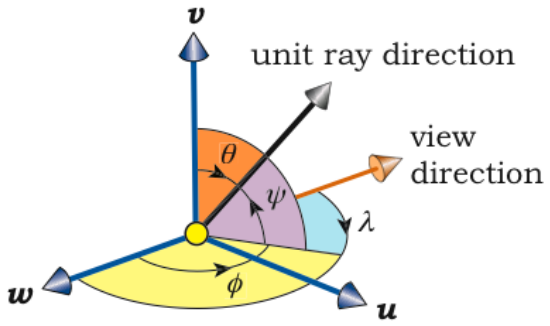
- This projection is very similar to the fisheye projection; except now both directions in the viewplane map a uniform angular distribution
- What shape has equal angular distances in all directions – a sphere!





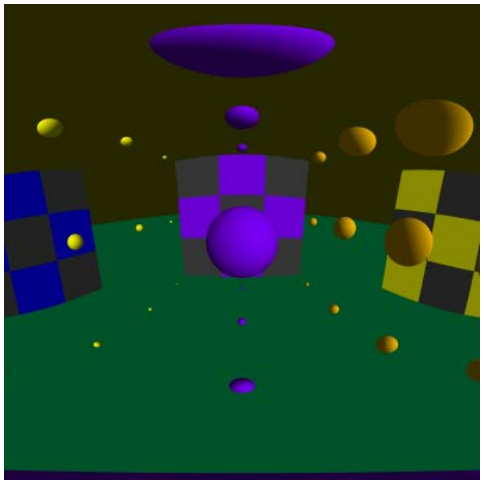
Spherical Panoramic Projection

- The view plane is equally divided by the angles
  - $\alpha = x_n \alpha_{max} \rightarrow \alpha \in [-\alpha, \alpha]$
  - $\beta = y_n \beta_{max} \rightarrow \beta \in [-\alpha/2, \alpha/2]$

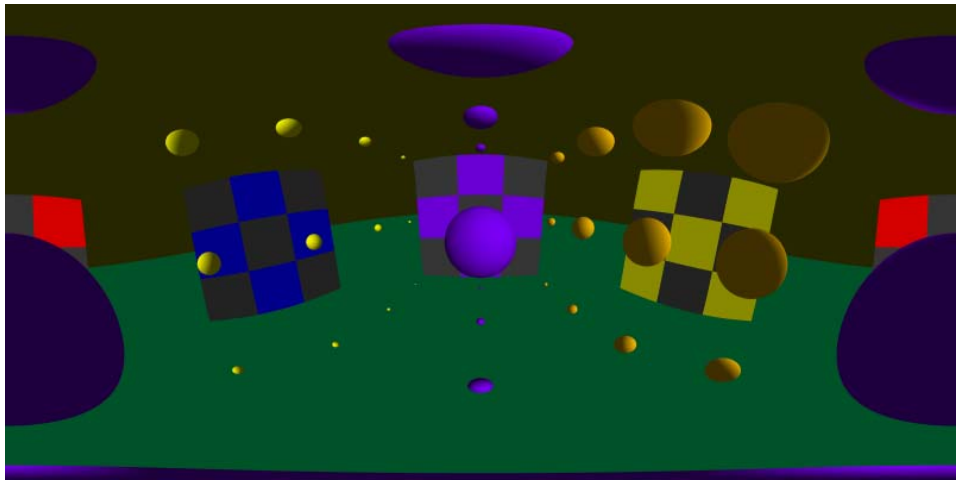


- Define the ray in terms of spherical coordinates
  - $\theta = \alpha - \beta$
  - $\phi = \alpha/2 \beta$
  - $\boldsymbol{d} = \sin\phi \sin\theta \, \boldsymbol{u} + \cos\phi \, \boldsymbol{v} + \sin\phi \cos\theta \, \boldsymbol{w}$
- Why isn't there a negative term like before?

Spherical Panoramic Examples



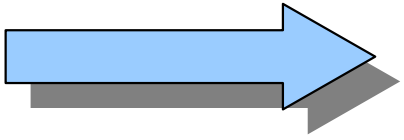
$\angle_{\mu\alpha\xi} = 90, \angle_{\mu\alpha\xi} = 90$

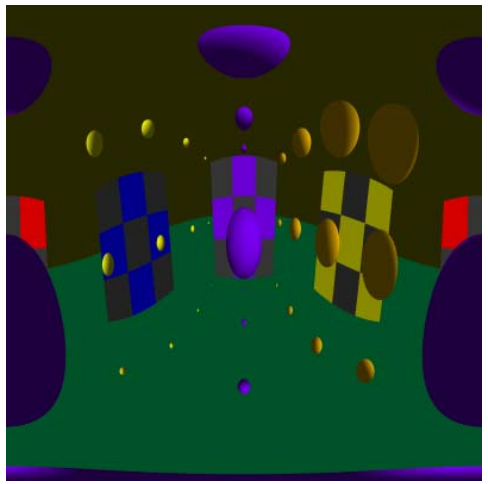


$\angle_{\mu\alpha\xi} = 180, \angle_{\mu\alpha\xi} = 90$

- Keeping the aspect ratio between the scene and the angles allowed the image to display more content. What happens if the aspect ratio isn't kept?

- The scene gets distorted. Twice as many vertical lines as horizontal lines.





Cylindrical Panoramic Projection

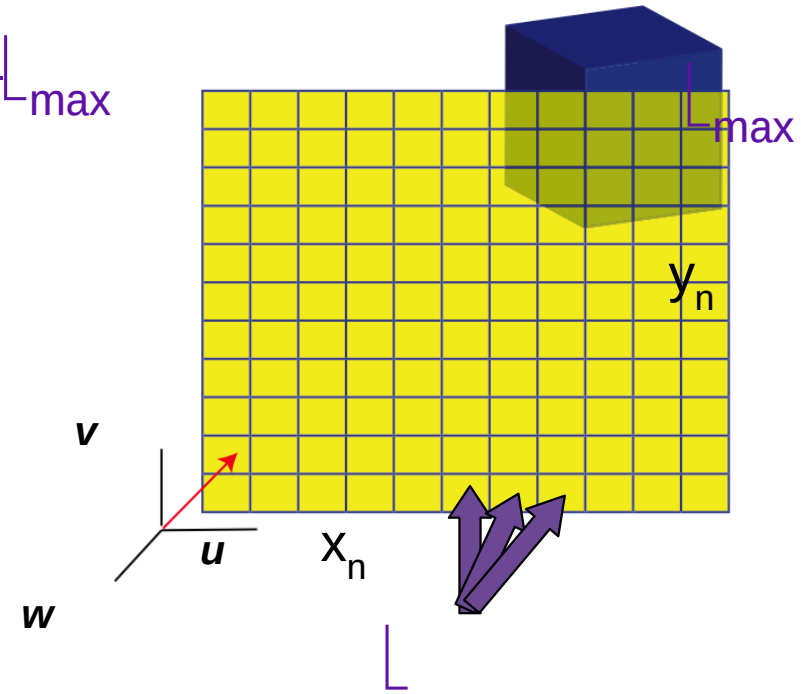
- A slight deviation from spherical projection is cylindrical projection.
- Similar to spherical coordinates the cylindrical projection covers 360° around the camera.
- The primary difference is it maintains the vertical direction.
  - The poles are not represented
  - As we approach the pole the scene stretch to infinity

spherical

cylindrical

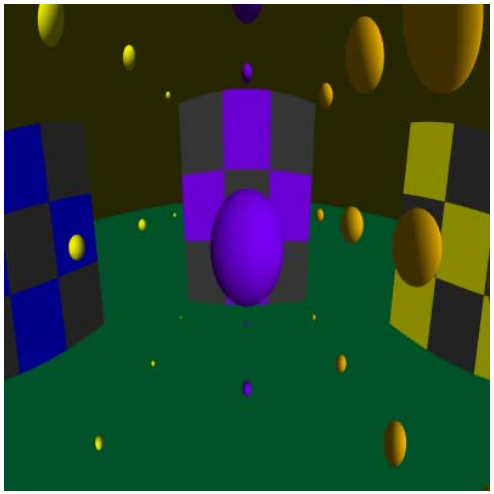
Cylindrical Panoramic Projection

- Again like the spherical projection the horizontal viewplane is divided up into equal angular distances.
  - $\alpha = x_n \alpha_{max} \rightarrow \alpha \in [-\beta, \beta]$
  - $\gamma = \beta \beta \alpha$
- Keep the vertical in normalized coordinates
- The ray direction equation becomes
  - $\boldsymbol{d} = \sin\gamma\, \boldsymbol{u} + y_n \boldsymbol{v} + \cos\gamma\, \boldsymbol{w}$

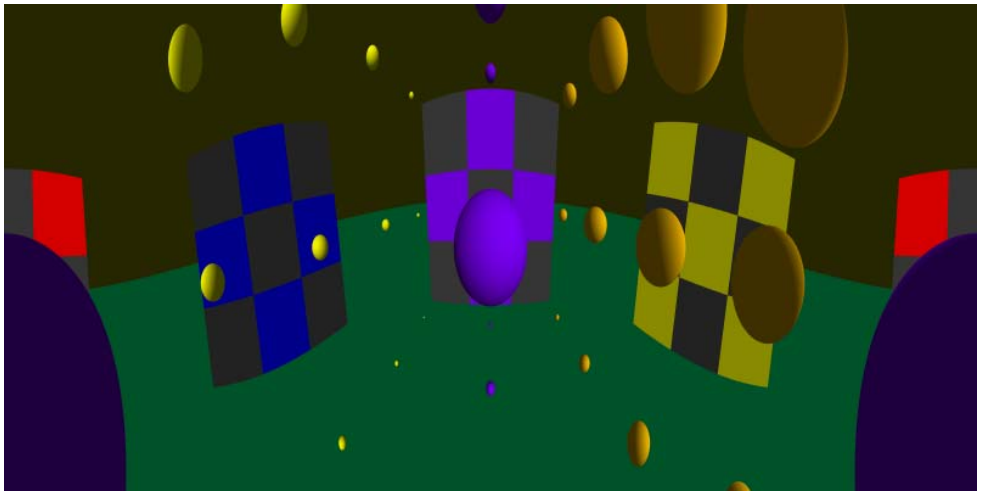


Cylindrical Panoramic Examples

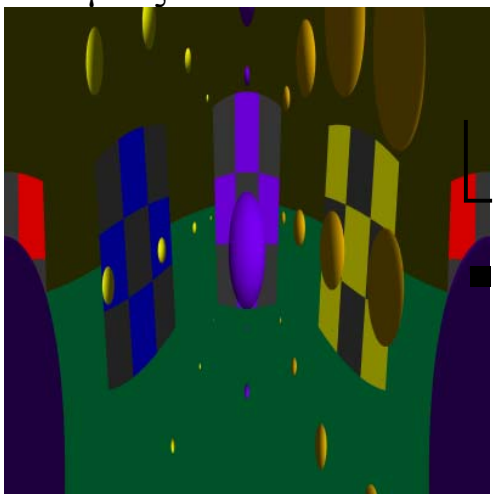
- Examples of the cylindrical projection



$\angle_{max} = 90$



$\angle_{max} = 180$



$\angle_{max} = 180$

- Resolution equal

How would you get more vertical?

References

- Strang, Gilbert (1998). Introduction to Linear Algebra, 2nd Edition. Pp 321-325 Wellesley, MA: Wellesley-Cambridge Press
- Suffern, Kevin (2007). Ray Tracing from the Ground Up. pp. 181-194 Wellesley, MA: A K Peters, Ltd.
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