CS 563 Advanced Topics in Computer Graphics
Direct Lighting - Path Tracing

Juan Li

Direct Lighting Path-Tracing Lighting

- Overview

The main idea.

- Theory

Endless Equation

- Implementation

PBRT Functions

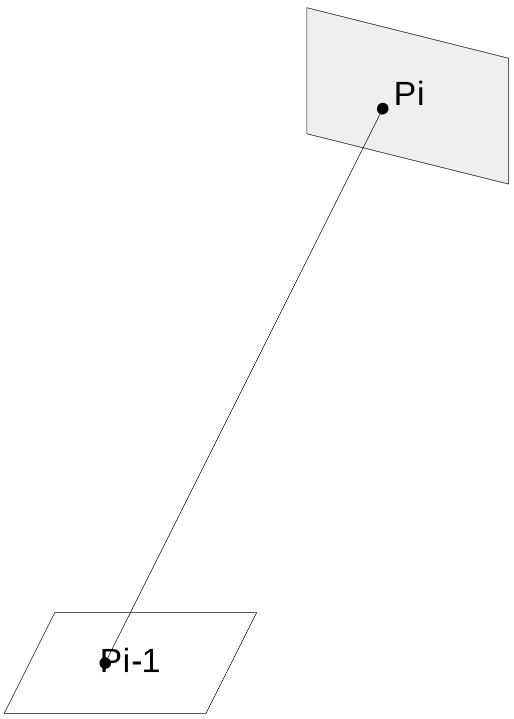
Overview

- What is Direct Lighting
- What is Path-Tracing Lighting
- Difference ?

Overview

- Direct Lighting

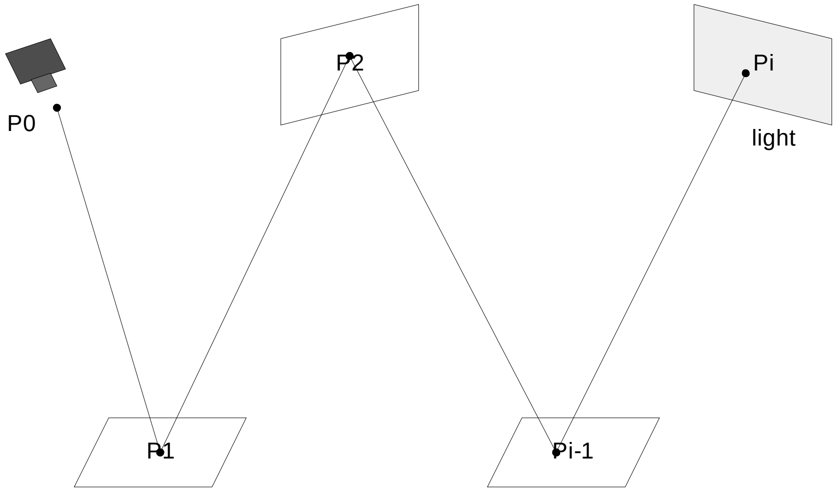
Only the light traveling directly from a light source to the shaded point accounts for shading.



Overview

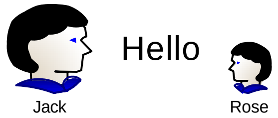
- Path-tracing Lighting

Tracing a path from the camera to light source. The shading of Pj in the path is the radiance along Pj to light source. Indirectly lighting.

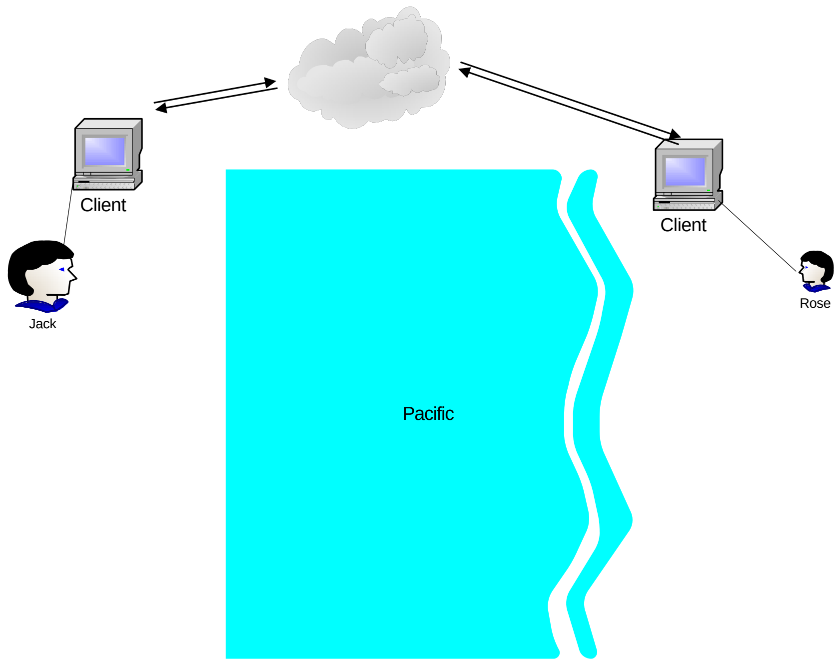


Overview

- Mutually visible. Get message directly



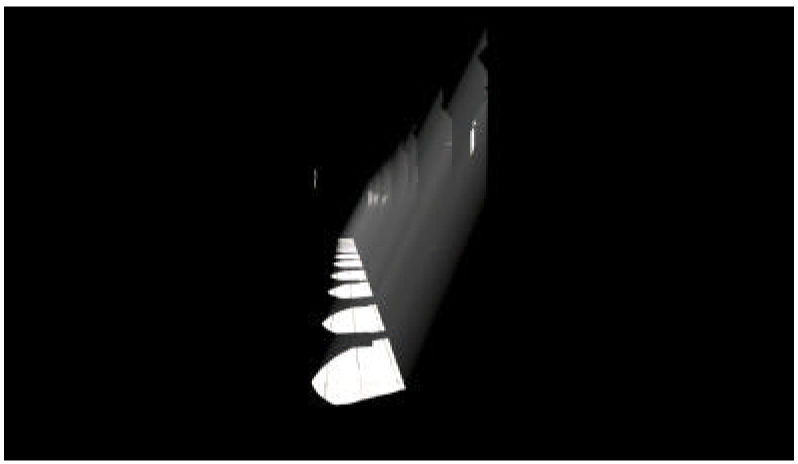
- Mutually invisible.Get message by path-tracing



Overview

- Direct Lighting

Path-tracing Lighting



Theory

- Balance Equation

The difference between the energy coming and going is equal to the difference between energy emitted and absorbed.

Outgoing-incoming=emission – absorbed

Since incoming-absorbed= reflected

$Lo(p,w_0) = L_e(p,w_0) + Lr(p,w_0)$

Theory

- Lighting quation

$Lo(p,w_0)=L_e(p,w_0)+?_s{}^2f(p,w_0,w_i)L_i\ (p,w_i)|cos?_i|dw_i;$

$Lo(p,w_0)$ is the exitant radiance from point p in direction $w_{0;}$

$f(p,w_0,w_i)$:BSDF for each direction $w_{i;}$

$L_i\ (p,w_i)$: incident radiance directly from light sources in DirectLighting.

Interested part for DirectLighting;

Theory

- Derivation to path-tracing

Remind LTE (light transport equation)

$Lo(p,w_0)=L_e(p,w_0)+?_s{}^2 f(p,w_0,w_i)L_i\,(p,w_i)|\cos?_i|dw_i;$

Assume no participation media, radiance is constant along rays.

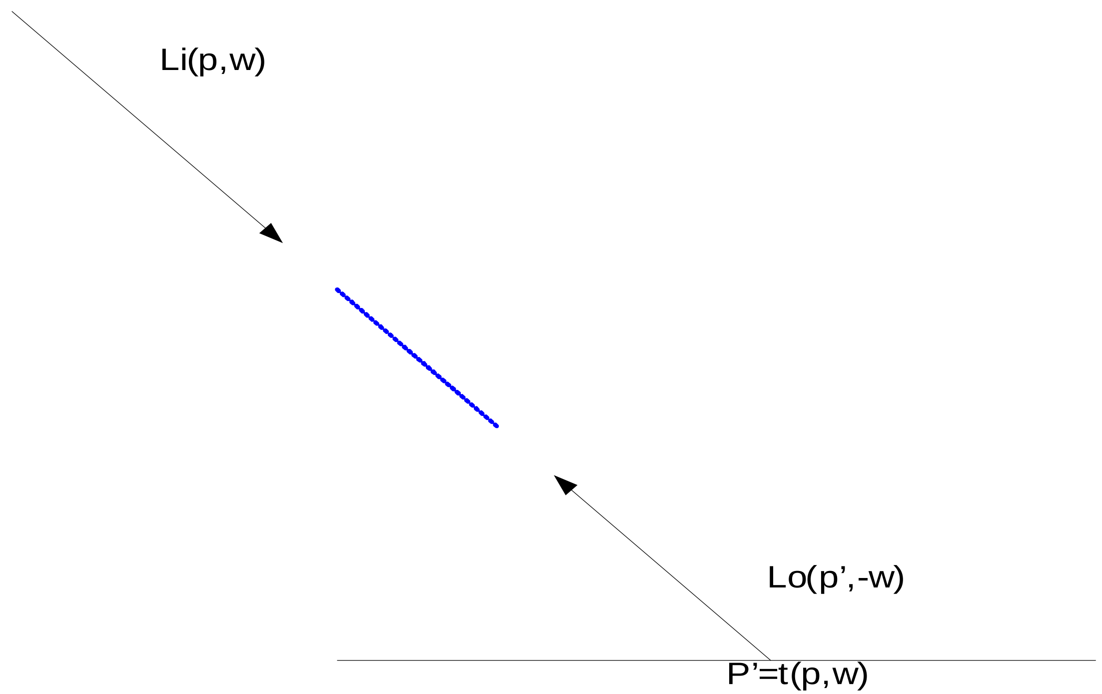
Then step1

$Lo(p,w_0)=L_e(p,w_0)+?_s{}^2 f(p,w_0,w_i)L_o\,(p',-w_i)|\cos?_i|dw_i;$

P' is the first surface point intersected by the ray from p with direction $w_{i;}$

Theory

- Radiance along a Ray through free space is unchanged. That means the incidance is equal to the exit of the first intersect point. Important!



Theory

- Step2

Rewrite equation in step1 as an integral over area instead of over directions on the sphere

1. Transform the direction w to points

L(p0,w)=L(p0->p1)

P0,p1 are mutually visible.

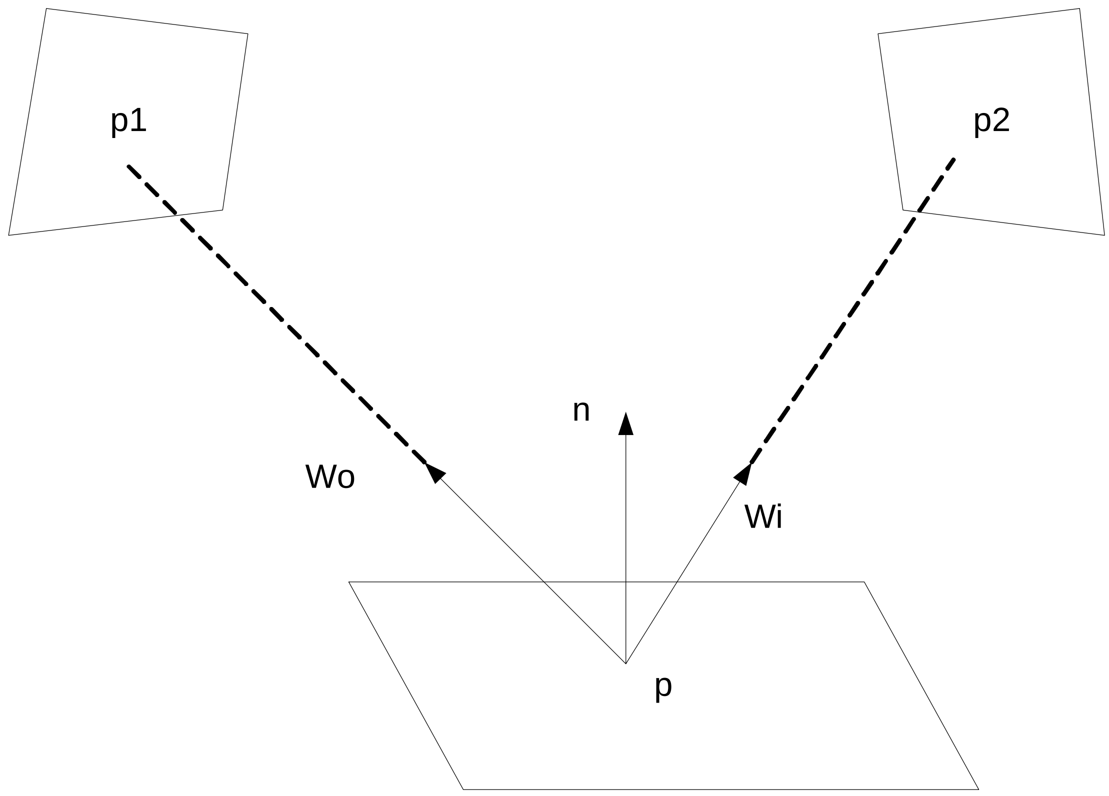
W = (p0->p1)

F(p0, w_0,w_i) = f(p2->p0->p1)

w_i=(p->p2)

w_0=(p->p1)

Theory



2.Transform the intergral over direction to over surface area with geometric coupling term $G(p<->p1)=V(p<->p1)|cos\ ?||cos\ ?_1|/||p-p1||^2$

V is binary visible function;

V=1 two points mutually visible.

V=0 otherwise;

- Derived equation from step 2

L(p->p1)=L_e(p->p1)+?$_A$f(p2->p->p1)L (p2->p)G(p2<->p)dA(p2);

Important step.

Difference from step1:

Step1: sample directions

Step2: Sample points on surfaces.

- Step3: recursively substitute $L(p_{i+1}\text{->}p_i)$ in the right side of the equation.

$L(p1\text{->}p0)=L_e(p1\text{->}p0)+?_A f(p2\text{->}p1\text{->}p0)\ L(p2\text{->}p1)\ G(p2\text{<->}p1)dA(p2);$

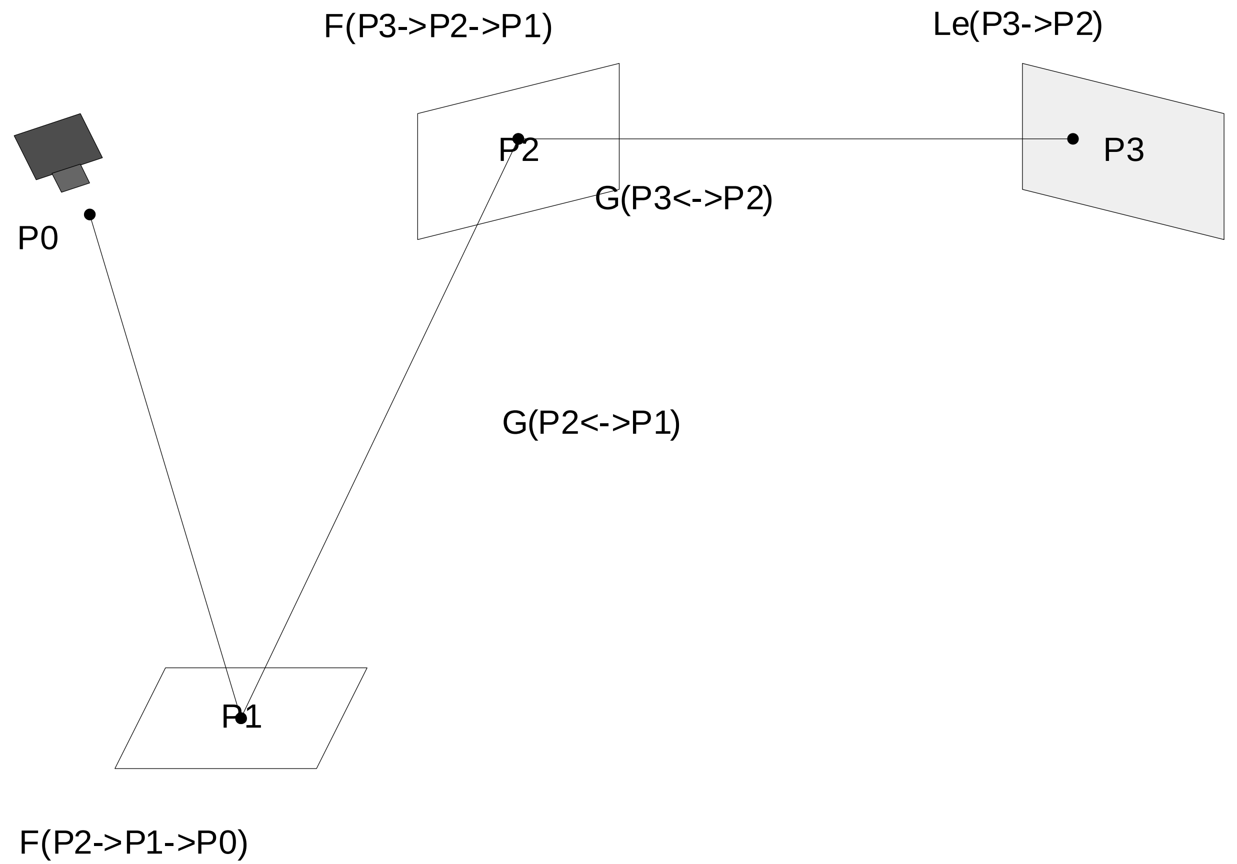
$= L_e(p1\text{->}p0)+?_A f(p2\text{->}p1\text{->}p0)\ (L_e(p2\text{->}p1)+?_A f(p3\text{->}p2\text{->}p1)\ L(p3\text{->}p2)\ G(p3\text{<->}p2)dA(p3))\ G(p2\text{<->}p1)dA(p2);$

....

$= L_e(p1\text{->}p0)+$

$?_A\ L_e(p2\text{->}p1)f(p2\text{->}p1\text{->}p0)\ G(p2\text{<->}p1)dA(p2)+$

$?_A\ ?_A\ L_e(p3\text{->}p2)f(p3\text{->}p2\text{->}p1)\ G(p3\text{<->}p2)\ f(p2\text{->}p1\text{->}p0)\ G(p2\text{<->}p1)dA(p3)dA(p2)+\ \dots$


F(P3->P2->P1)
Le(P3->P2)
P2
P3
G(P3<->P2)
P0
G(P2<->P1)
P1
F(P2->P1->P0)

- This infinite sum can be written as

L(p1->p0)= $\sum_{i=1}^{\infty} P(\overline{p_i})$

$$P(\overline{p_i}) = \int_A \int_A \ldots \int_A L_e(p_i \to p_{i-1}) \left(\prod_{j=1}^{i-1} f(p_{j+1} \to p_j \to p_{j-1}) G(p_{j+1} \leftrightarrow p_j) \right) dA(p_2) \ldots dA(p_i)$$

- Problems

1. How to estimate the value of the sum of the infinite number of P($\underline{p}_i$)
2. For a particular P($\underline{p}_i$) how to generate the path $\underline{p}$.

- Soulutions

1. Using Russian roulette to stop sampling after a finite number of terms.

$$P(\overline{p}_1)+P(\overline{p}_2)+P(\overline{p}_3)+\frac{1}{1-q}\sum_{i=4}^{\infty}P(\overline{p}_i)$$

   - Stop with probability q

- 2. incremental path construction

Constructs the path incrementally.

From camera p_0

For p_i sample w_i, trace a ray from p_i with w_i and find the closest intersection.