CS/EE 545 NAME:
Digital Image Processing Fall Semester 2002

Examination #2

DO NOT OPEN THIS EXAMINATION UNTIL YOU ARE TOLD TO DO SO!

Write your name at the top of this page now.

This examination is OPEN BOOK and OPEN NOTES.

Write all your answers on the examination in the space provided. You may use the back of the examination for extra space. Partial credit will be given, but you must justify your work. If you do not understand a question, ask. It will be to your advantage to read the entire examination before beginning to work.

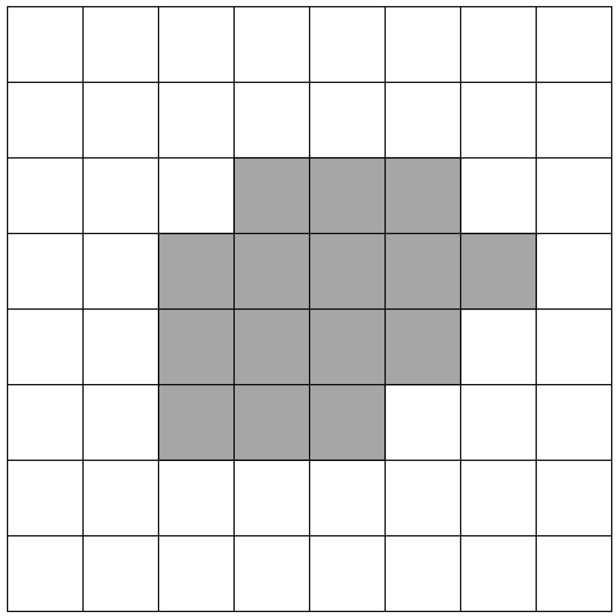
The examination will end exactly 90 minutes after it begins. Good luck.

Problem 1:	/30
Problem 2:	/20
Problem 3:	/35
Problem 4:	/15
Total:	/100

PROBLEM 1 (30 Points)
Define the *dirosion* of binary image A by structuring element B as the set difference of the dilation of A by B and the erosion of A by B. That is,

$$\text{dirosion}(A, B) = (A \oplus B) - (A \ominus B)$$

Part A (15 Points)
Suppose that binary image A is given by



Indicate on the figure the dirosion of A by structuring element

$$B_1 = \{(-1, 0), (0, 0), (1, 0), (0, -1), (0, 1)\}.$$

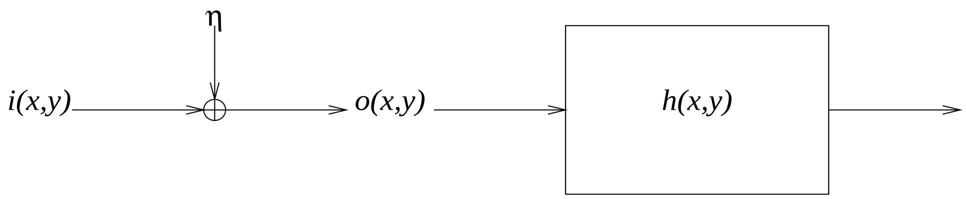
Part B (5 Points)
Describe in English what the dirosion operation does.

Part C (10 Points)
Evaluate and discuss the suitability of using structuring element

$$B_2 = \{(-1, -1), (0, -1), (1, -1), (-1, 0), (0, 0), (1, 0), (-1, 1), (0, 1), (1, 1)\}$$

for dirosion.

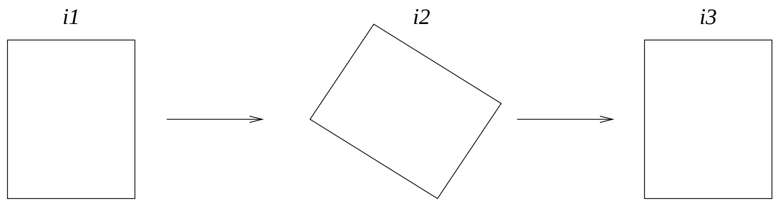
PROBLEM 2 (20 Points)
Suppose that noise $\eta(x, y)$ is added to image $i(x, y)$ to produce output $o(x, y)$. It is desired to filter o with an optimal (Wiener) filter $h(x, y)$ to produce the best possible estimate of i as shown in the block diagram.



If the noise has exactly the same power spectrum as the signal, what is the impulse response of the optimal filter h?

PROBLEM 3 (35 Points)
Image $i_1(x,y)$ is rotated about its center by amount θ to produce image $i_2(x,y)$. Each point of $i_2(x,y)$ is obtained by 4-point interpolation from $i_1(x,y)$. That is, one finds the point (x',y') in i_1 which, when rotated by θ, maps to (x,y) in i_2. Because (x',y') will not generally be integers, one finds the 4 closest points in i_1 and interpolates from them using, e.g., bilinear interpolation.

Image i_2 is rotated about its center by amount $-\theta$ to produce image i_3, which ought to be similar, but not identical, to i_1.



One can view this as a sort of transformation h_θ, given by

$$i_3(x,y) = \sum_{x'=0}^{N-1} \sum_{y'=0}^{N-1} h_\theta(x,y;x',y') i_1(x',y')$$

Note that we use to h_θ to denote both the operation and the kernel of the operation.

Answer the following questions, *providing justifications for each answer*. Do not assume that θ takes on any special values.

Part A (5 Points)
Is h_θ a linear operation?

Part B (5 Points)
Is h_θ a spatially invariant or spatially varying operation?

Part C (10 Points)
At most, how many pixels in i_1 contribute to each pixel in i_3?

Part D (5 Points)
At most, how many pixels in i_3 depend on each pixel in i_1?

Part E (5 Points)
Can h_θ be implemented as a convolution?

Part F (5 Points)
Does the Fourier Transform of h_θ exist?

PROBLEM 4 (15 Points)
Let $f(x, y)$ be an image with DFT $F(u, v)$. Assume that f has dimensions $N \times N$ where N is a power of 2, that is, $N = 2^K$ for some K. Consider the Discrete Wavelet Transform $W(u, v)$ of $F(u, v)$ using the Haar basis functions.

Part A (5 Points)
Let the DWT be based on the maximum number of scales, which is K. What significance can you attach to the value of $W(0, 0)$?

Hint: Consider how the sum of all values of $F(u, v)$ relates to $f(x, y)$.

Part B (10 Points)
In an effort to denoise the image, $W(u, v)$ is thresholded at T to produce $W'(u, v)$.

$$W'(u,v) = \begin{cases} W(u,v), & \text{if } |W(u,v)| \geq T; \\ 0, & \text{otherwise.} \end{cases}$$

The inverse DWT of $W'(u, v)$ is computed to give $F'(u, v)$ and the inverse DFT taken to produce to $f'(x, y)$.

Evaluate this scheme.