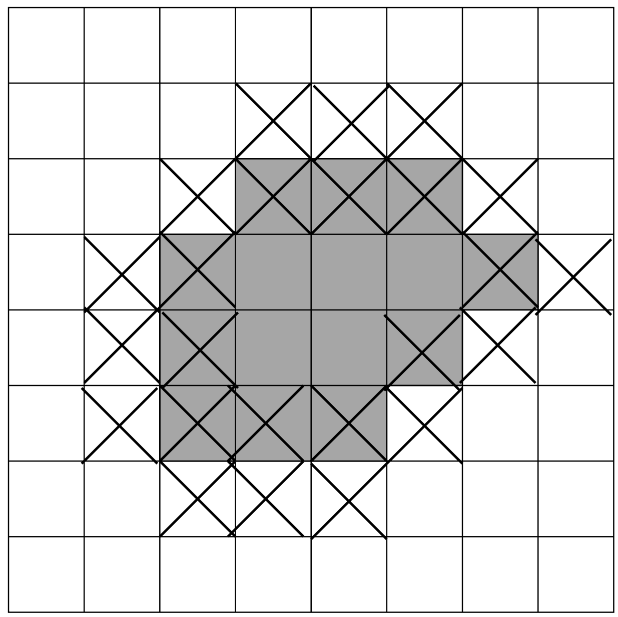
CS/EE 545
Digital Image Processing — Fall Semester 2002

Examination #2 Solutions

PROBLEM 1 (30 Points)
Define the *dirosion* of binary image A by structuring element B as the set difference of the dilation of A by B and the erosion of A by B. That is,

$$\text{dirosion}(A, B) = (A \oplus B) - (A \ominus B)$$

Part A (15 Points)
Suppose that binary image A is given by



Indicate on the figure the dirosion of A by structuring element

$$B_1 = \{(-1, 0), (0, 0), (1, 0), (0, -1), (0, 1)\}.$$

The dirosion is indicated by the Xs on the figure.

Part B (5 Points)
Describe in English what the dirosion operation does.

It detects region edges and/or identifies region boundaries.

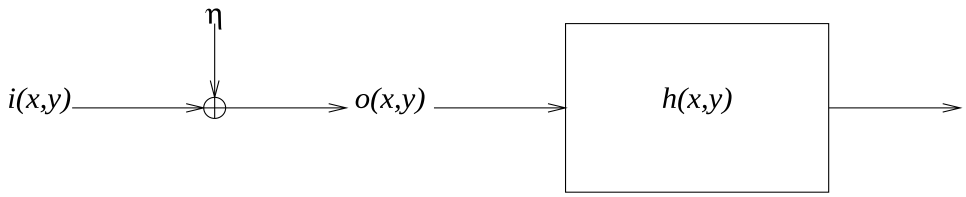
Part C (10 Points)
Evaluate and discuss the suitability of using structuring element

$$B_2 = \{(-1, -1), (0, -1), (1, -1), (-1, 0), (0, 0), (1, 0), (-1, 1), (0, 1), (1, 1)$$

for dirosion.

This structuring element has the good properties of being compact and rotationally symmetric. However, its larger shape compared wih B_1 means that it will produce thicker boundaries, especially at corners.

PROBLEM 2 (20 Points)
Suppose that noise $\eta(x, y)$ is added to image $i(x, y)$ to produce output $o(x, y)$. It is desired to filter o with an optimal (Wiener) filter $h(x, y)$ to produce the best possible estimate of i as shown in the block diagram.



If the noise has exactly the same power spectrum as the signal, what is the impulse response of the optimal filter h?

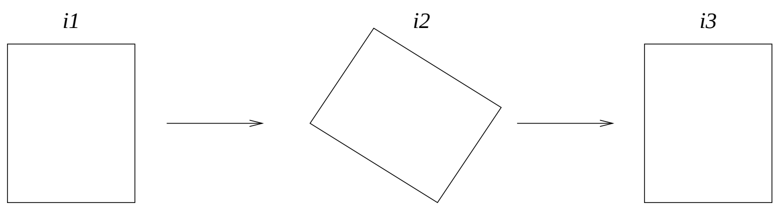
The optimal filter has frequency response given by

$$\hat{H} = \frac{H^*(u,v)}{|H(u,v)|^2 + \Phi_{\eta\eta}(u,v)/\Phi_{ff}(u,v)}$$

Because there is no degradation, $H = 1$ and because the noise and signal have the same power spectrum, $\Phi_{\eta\eta} = \Phi_{ff}$. Therefore, $\hat{H}(u,v) = 1/2$ and $h(x,y) = \delta(x,y)/2$.

PROBLEM 3 (35 Points)
Image $i_1(x,y)$ is rotated about its center by amount θ to produce image $i_2(x,y)$. Each point of $i_2(x,y)$ is obtained by 4-point interpolation from $i_1(x,y)$. That is, one finds the point (x', y') in i_1 which, when rotated by θ, maps to (x,y) in i_2. Because (x', y') will not generally be integers, one finds the 4 closest points in i_1 and interpolates from them using, e.g., bilinear interpolation.

Image i_2 is rotated about its center by amount $-\theta$ to produce image i_3, which ought to be similar, but not identical, to i_1.



One can view this as a sort of transformation h_θ, given by

$$i_3(x,y) = \sum_{x'=0}^{N-1} \sum_{y'=0}^{N-1} h_\theta(x,y;x',y') i_1(x',y')$$

Note that we use to h_θ to denote both the operation and the kernel of the operation.

Answer the following questions, *providing justifications for each answer.* Do not assume that θ takes on any special values.

Part A (5 Points)
Is h_θ a linear operation?

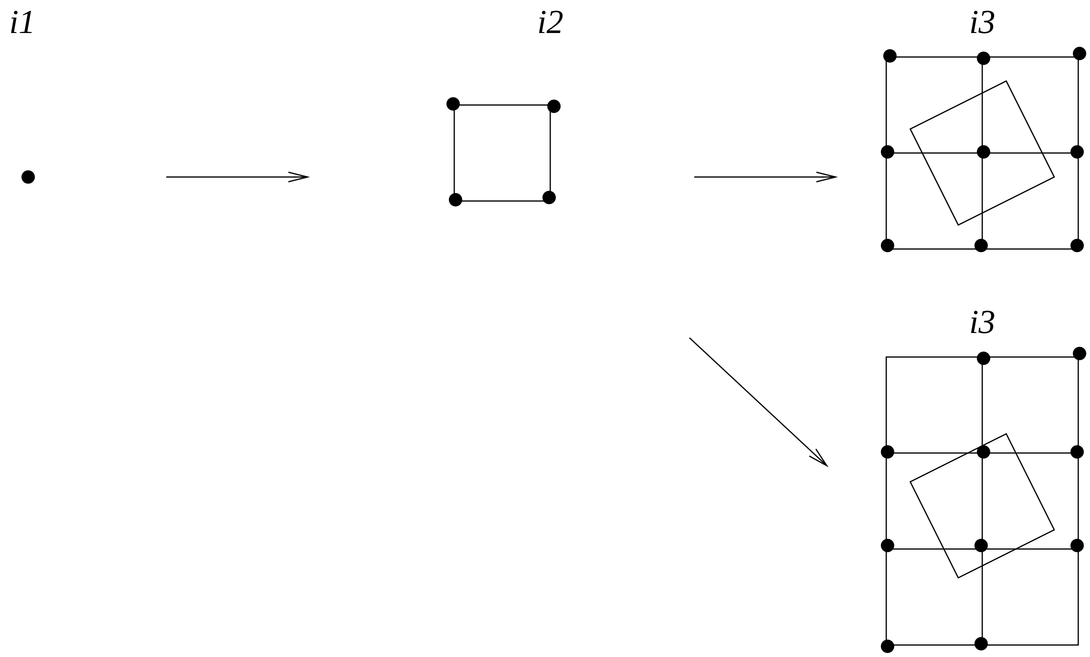
Yes, it is linear. Multiplying i_1 by a constant multiplies 1_2 by the same constant and adding 2 images produces the sum of the rotated images.

Part B (5 Points)
Is h_θ a spatially invariant or spatially varying operation?

h_θ is spatially varying. $i_2(0,0) = i_1(0,0)$, but output pixels farther from the origin may depend on more than 1 input pixel.

Part C (10 Points)
At most, how many pixels in i_1 contribute to each pixel in i_3?

10. A pixel in i_1 may contribute to as many as 4 pixels in i_2, but cannot contribute to 16 pixels in i_3. See the figure for how a pixel in i_1 can contribute to 9 or 10 pixels in i_3.



Part D (5 Points)
At most, how many pixels in i_3 depend on each pixel in i_1?

Same as Part C.

Part E (5 Points)
Can h_θ be implemented as a convolution?

No, it is linear but not shift-invariant.

Part F (5 Points)
Does the Fourier Transform of h_θ exist?

No. Same reason as Part E.

PROBLEM 4 (15 Points)
Let $f(x, y)$ be an image with DFT $F(u, v)$. Assume that f has dimensions $N \times N$ where N is a power of 2, that is, $N = 2^K$ for some K. Consider the Discrete Wavelet Transform $W(u, v)$ of $F(u, v)$ using the Haar basis functions.

Part A (5 Points)
Let the DWT be based on the maximum number of scales, which is K. What significance can you attach to the value of $W(0, 0)$?

Hint: Consider how the sum of all values of $F(u, v)$ relates to $f(x, y)$.

$f(0, 0) = \sum F(u, v)$. $W(0, 0)$ is the average value of $F(u, v)$ and is therefore equal to $f(0, 0)/N^2$. This answer could change slightly depending on what constants are used for the Haar wavelets.

Part B (10 Points)
In an effort to denoise the image, $W(u, v)$ is thresholded at T to produce $W'(u, v)$.

$$W'(u,v) = \begin{cases} W(u,v), & \text{if } |W(u,v)| \geq T; \\ 0, & \text{otherwise.} \end{cases}$$

The inverse DWT of $W'(u, v)$ is computed to give $F'(u, v)$ and the inverse DFT taken to produce to $f'(x, y)$.

Evaluate this scheme.

As T gets larger, the result gets closer to the orginal image. As T approaches 0, the result becomes less like the original. In between, it is hard to say what will happen. The difficulty is that the components of the DFT may not be correlated over similar wavelengths, unlike the spatial image components, which do tend to be correlated at similar positions. Thus, one would not necessarily expect the noise to be filtered out by thesholding the DWT of the DFT.