CS/EE 545 NAME:
Digital Image Processing Fall Semester 2002

Examination #1

DO NOT OPEN THIS EXAMINATION UNTIL YOU ARE TOLD TO DO SO!

Write your name at the top of this page now.

This examination is OPEN BOOK and OPEN NOTES.

Write all your answers on the examination in the space provided. You may use the back of the examination for extra space. Partial credit will be given, but you must justify your work. If you do not understand a question, ask. It will be to your advantage to read the entire examination before beginning to work.

The examination will end exactly 90 minutes after it begins. Good luck!

Problem 1: /40

Problem 2: /20

Problem 3: /20

Problem 4: /20

Total: /100

PROBLEM 1 (40 Points)
Consider the following convolutional kernel $L(x, y)$:

0	1/4	0
1/4	-1	1/4
0	1/4	0

Part A (10 Points)
Is L separable into a product of an x-only kernel and a y-only kernel? Show what they are or explain why this is impossible.

Part B (10 Points)
Is L separable into a sum of an x-only kernel and a y-only kernel? Show what they are or explain why this is impossible.

Part C (20 Points)
Assuming that L is in an $N \times N$ image, show that the DFT of L is proportional to

$$L(u,v) = -1 + \cos\left(\frac{\pi}{N}(u+v)\right)\cos\left(\frac{\pi}{N}(u-v)\right).$$

Hint: Recall that $\cos(A) + \cos(B) = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$.

PROBLEM 2 (20 Points)
The discrete correlation of two signals $a(x, y)$ and $b(x, y)$ is given by

$$\phi_{ab}(x, y) = a * b = \sum_{x'=0}^{M-1} \sum_{y'=0}^{N-1} a(x' - x, y' - y) b(x', y')$$

Note that the correlation differs from the convolution in that convolution would use $a(x - x', y - y')$. Show that

$$\phi_{ab}(x, y) = \phi_{ba}(-x, -y)$$

assuming that images a and b wrap around when $x \geq M$ or $y \geq N$. That is, $a(x + M, y) = a(x, y + N) = a(x, y)$. Similarly for b.

PROBLEM 3 (20 Points)
An image $f(x, y)$ is binarized to produce black-and-white image $b(x, y)$ which has equal numbers of black pixels (graylevel 0) and white pixels (graylevel 255).

Image processing expert Dr. Ima Ging passes $b(x, y)$ through two filters:

- A 3×3 blurring filter with coefficients equal to $\frac{1}{5}$ as shown,

0	1/5	0
1/5	1/5	1/5
0	1/5	0

  and

- A 3×3 median filter.

to produce output $o(x, y)$.

Part A (10 Points)
Assuming that the blurring filter is applied before the median filter, what are the possible output pixel values? Hint: Not all 255 possibilities can occur.

Part B (10 Points)
Unfortunately, Ima has quite forgotten in which order the two filtering operations were performed. Give a binary image function $b(x, y)$ for which $o(x, y)$ would look the same no matter in which order the filters are applied.

PROBLEM 4 (20 Points)
In his implementation of the forward and inverse DFT, Dr. Otto Korrelation switched the signs in the exponents. That is, his programs compute

$$F'(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{+i2\pi(ux/M+vy/N)}, \text{and } f'(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F'(u,v)e^{-i2\pi(ux/M+vy/N)}$$

Part A (10 Points)
Dr. Korrelation takes an image $f(x,y)$, computes the incorrect DFT $F'(u,v)$ and then computes the incorrect inverse DFT $f'(x,y)$. Express $f'(x,y)$ in terms of $f(x,y)$. Explain.

Part B (10 Points)
For what input images $f(x,y)$ would Dr. Korrelation's incorrect DFT $F'(u,v)$ be equal to the correct DFT $F(u,v)$. Give necessary and sufficient conditions on f.