CS/EE 545
Digital Image Processing Fall Semester 2002

Examination #1 Solutions

PROBLEM 1 (40 Points)
Consider the following convolutional kernel $L(x, y)$:

0	1/4	0
1/4	-1	1/4
0	1/4	0

Part A (10 Points)
Is L separable into a product of an x-only kernel and a y-only kernel? Show what they are or explain why this is impossible.

No. Suppose that L were the product of $[x_0, x_1, x_2]$ and $[y_0, y_1, y_2]$ Then $x_0 = 0$ or $y_0 = 0$ because $L_{00} = x_0 y_0 = 0$. But if that were true, then either the entire top row or the entire left column of L should be 0, depending on whether $x_0 = 0$ or $y_0 = 0$. Since this is not the case, L must not be separable.

Part B (10 Points)
Is L separable into a sum of an x-only kernel and a y-only kernel? Show what they are or explain why this is impossible.

The x-only and y-only kernels are $[1/4, -1/2, 1/4]$.

Part C (20 Points)
Assuming that L is in an $N \times N$ image, show that the DFT of L is proportional to

$$L(u, v) = -1 + \cos\left(\frac{\pi}{N}(u + v)\right) \cos\left(\frac{\pi}{N}(u - v)\right).$$

$$\begin{aligned}
L(u, v) &= \frac{1}{N} \Sigma_x \Sigma_y L(x, y) e^{-2\pi i(ux+vy)/N} \\
&= \frac{1}{N}\left(-1 + \frac{1}{4}\left(e^{2\pi iu/N} + e^{-2\pi iu/N} + e^{-2\pi iv/N} + e^{2\pi iv/N}\right)\right) \\
&= \frac{1}{N}\left(-1 + \frac{1}{2}\left(\cos(2\pi u/n) + \cos(2\pi v/N)\right)\right) \\
&= \frac{1}{N}\left(-1 + \cos(\pi(u + v)/n) + \cos(\pi(u - v)/N)\right)
\end{aligned}$$

PROBLEM 2 (20 Points)
The discrete correlation of two signals $a(x, y)$ and $b(x, y)$ is given by

$$\phi_{ab}(x, y) = a * b = \sum_{x'=0}^{M-1} \sum_{y'=0}^{N-1} a(x' - x, y' - y) b(x', y')$$

Note that the correlation differs from the convolution in that convolution would use $a(x - x', y - y')$. Show that

$$\phi_{ab}(x, y) = \phi_{ba}(-x, -y)$$

assuming that images a and b wrap around when $x \geq M$ or $y \geq N$. That is, $a(x + M, y) = a(x, y + N) = a(x, y)$. Similarly for b.

Let $x'' = x' - x$, $y'' = y' - y$. Then

$$\begin{aligned}
\phi_{ab}(x,y) &= \sum_{x'=0}^{M-1}\sum_{y'=0}^{N-1} a(x'-x, y'-y)b(x',y') \\
&= \sum_{x''=-x}^{M-x-1}\sum_{y''=-y}^{N-y-1} a(x'',y'')b(x''+x, y'+y) \\
&= \sum_{x''=0}^{M-1}\sum_{y''=0}^{N-1} a(x'',y'')b(x''+x, y'+y) \\
&= \sum_{x''=0}^{M-1}\sum_{y''=0}^{N-1} b(x''+x, y'+y)a(x'',y'') \\
&= \phi_{ba}(-x,-y)
\end{aligned}$$

PROBLEM 3 (20 Points)
An image $f(x,y)$ is binarized to produce black-and-white image $b(x,y)$ which has equal numbers of black pixels and white pixels. The binary image is slightly scaled, so that the black pixels have value 0, but the white pixels have value 252.

Image processing expert Dr. Ima Ging passes $b(x,y)$ through two filters:

- A 3×3 blurring filter with coefficients equal to $\frac{1}{5}$ as shown,

0	1/5	0
1/5	1/5	1/5
0	1/5	0

  and

- A 3×3 median filter.

to produce output $o(x,y)$.

Part A (10 Points)
Assuming that the blurring filter is applied before the median filter, what are the possible output pixel values? Hint: Not all 255 possibilities can occur.

Possible values are:

$$0, 51, 102, 153, 204, 255$$

depending on how many of a pixel's 4 up/down/left/right neighbors (and itself) are black or white.

Part B (10 Points)
Unfortunately, Ima has quite forgotten in which order the two filtering operations were performed. Give a binary image function $b(x,y)$ for which $o(x,y)$ would look the same no matter in which order the filters are applied.

$$b(x, y) = \begin{cases} 0, & \text{if } x < N; \\ 255, & \text{otherwise.} \end{cases}$$

for example. In this case, the median filtering does nothing. There are many other possible answers.

PROBLEM 4 (20 Points)
In his implementation of the forward and inverse DFT, Dr. Otto Korrelation switched the signs in the exponents. That is, his programs compute

$$F'(u, v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{+i2\pi(ux/M+vy/N)}, \text{and } f'(x, y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F'(u, v) e^{-i2\pi(ux/M+vy/N)}$$

Part A (10 Points)
Dr. Korrelation takes an image $f(x, y)$, computes the incorrect DFT $F'(u, v)$ and then computes the incorrect inverse DFT $f'(x, y)$. Express $f'(x, y)$ in terms of $f(x, y)$. Explain.

Because of the sign switches, $F'(u, v) = F(-u, -v)$, but $f'(x, y) = IFT\{F'(-u, -v)\} = IFT\{F', v)\} = f(x, y)$. That is, $f' = f$.

Part B (10 Points)
For what input images $f(x, y)$ would Dr. Korrelation's incorrect DFT $F'(u, v)$ be equal to the correct DFT $F(u, v)$. Give necessary and sufficient conditions on f.

As long as f is an even and real function of x and y, $F(u, v)$ will be an even and real function of u and v, and $F'(u, v) = F(-u, -v) = F(u, v)$ will hold.