















































Clustering: Data Scaling (1 of 3)

- Final results depend upon relative ranges Typically scale so relative ranges equal
- Normalize to Zero Mean and Unit Variance
 - Mean x_k , stddev s_k of the k^{th} parameter

$$x'_{ik} = \frac{x_{ik} - \bar{x}_k}{s_k}$$

Do this for each of the k paramters



Clustering: Data Scaling (2 of 3)

- Weights
 - $x'_{ik} = w_k x_{ik}$
- Assign based on relative importance
- Range Normalization
 - Change from $[x_{min,k},x_{max,k}]$ to [0,1]

$$x'_{ik} = \frac{x_{ik} - x_{min,k}}{x_{max,k} - x_{min,k}}$$

- Ex: x_{i1} {1, 6, 5, 11}
 - 1→0, 11→1, 6→ .5, 4→.4
- Sensitive to outliers (say 11 above was 101)

Clustering: Data Scaling (3 of 3)

- Percentile Normalization
 - Scale so 95% of values between 0 and 1

$$x'_{ik} = \frac{x_{ik} - x_{2.5,k}}{x_{97.5,k} - x_{2.5,k}}$$

Less sensitive to outliers



Clustering: Distance Metric (1 of 2)

- Map each component to n-dimensional space and see which are close to each other
- Euclidean Distance between two components
 - $\{x_{i1}, x_{i2}, ... x_{in}\}$ and $\{x_{i1}, x_{i2}, ..., x_{in}\}$

$$d = \left\{ \sum_{k=1}^{n} (x_{ik} - x_{jk})^2 \right\}^{0.5}$$

- Weighted Euclidean Distance
 - Assign weights a_k for n parameters
 - * Used if values not scaled or if significantly different in importance $d = \sum\limits_{k=1}^n \left\{ a_k (x_{ik} x_{jk})^2 \right\}^{0.5}$

$$d = \sum_{k=1}^{n} \left\{ a_k (x_{ik} - x_{jk})^2 \right\}^{0.5}$$



Clustering: Distance Metric (2 of 2)

- Chi-Square Distance:
 - Used in distribution fitting
 - Need to use normalized or relative sizes influence distance

$$d = \sum_{k=1}^{n} \left\{ \frac{(x_{ik} - x_{jk})^2}{x_{ik}} \right\}$$

Overall, Euclidean distance is most commonly used



Clustering Methodology

- 1. Take sample
- 2. Select parameters
- 3. Transform, if necessary
- 4. Remove outliers
- 5. Scale observations
- 6. Select distance metric
- 7. Perform clustering
- 8. Interpret
- 9. Change and repeat 3-7
- 10. Select representative components



















